on the problems in "7 Inverse hyperbolic functions/7.1 Inverse hyperbolic sine"
Test results for the 46 problems in "7.1.2 (dx)^m (a+b arcsinh(c x))^n.txt"
Problem 24: Unable to integrate problem.

$$
\int x^{4} \operatorname{arcsinh}(a x)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 4, 202 leaves, 41 steps):
$\frac{x^{5} \operatorname{arcsinh}(a x)^{3 / 2}}{5}+\frac{3 \operatorname{erf}(\sqrt{5} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{5} \sqrt{\pi}}{16000 a^{5}}+\frac{3 \operatorname{erfi}(\sqrt{5} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{5} \sqrt{\pi}}{16000 a^{5}}-\frac{\operatorname{erf}(\sqrt{3} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{3} \sqrt{\pi}}{384 a^{5}}$
$-\frac{\operatorname{erfi}(\sqrt{3} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{3} \sqrt{\pi}}{384 a^{5}}+\frac{3 \operatorname{erf}(\sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{64 a^{5}}+\frac{3 \operatorname{erfi}(\sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{64 a^{5}}-\frac{4 \sqrt{a^{2} x^{2}+1} \sqrt{\operatorname{arcsinh}(a x)}}{25 a^{5}}$
$+\frac{2 x^{2} \sqrt{a^{2} x^{2}+1} \sqrt{\operatorname{arcsinh}(a x)}}{25 a^{3}}-\frac{3 x^{4} \sqrt{a^{2} x^{2}+1} \sqrt{\operatorname{arcsinh}(a x)}}{50 a}$
Result(type 8, 12 leaves):

$$
\int x^{4} \operatorname{arcsinh}(a x)^{3 / 2} \mathrm{~d} x
$$

Problem 25: Unable to integrate problem.

$$
\int x^{3} \operatorname{arcsinh}(a x)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 4, 149 leaves, 25 steps):
$-\frac{3 \operatorname{arcsinh}(a x)^{3 / 2}}{32 a^{4}}+\frac{x^{4} \operatorname{arcsinh}(a x)^{3 / 2}}{4}+\frac{3 \operatorname{erf}(\sqrt{2} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{2} \sqrt{\pi}}{256 a^{4}}-\frac{3 \operatorname{erfi}(\sqrt{2} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{2} \sqrt{\pi}}{256 a^{4}}-\frac{3 \operatorname{erf}(2 \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{2048 a^{4}}$
$+\frac{3 \operatorname{erfi}(2 \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{2048 a^{4}}+\frac{9 x \sqrt{a^{2} x^{2}+1} \sqrt{\operatorname{arcsinh}(a x)}}{64 a^{3}}-\frac{3 x^{3} \sqrt{a^{2} x^{2}+1} \sqrt{\operatorname{arcsinh}(a x)}}{32 a}$
Result(type 8, 12 leaves):

$$
\int x^{3} \operatorname{arcsinh}(a x)^{3 / 2} \mathrm{~d} x
$$

Problem 27: Unable to integrate problem.

$$
\int \frac{x^{4}}{\sqrt{\operatorname{arcsinh}(a x)}} \mathrm{d} x
$$

Optimal(type 4, 119 leaves, 18 steps):
$\frac{\operatorname{erf}(\sqrt{5} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{5} \sqrt{\pi}}{160 a^{5}}+\frac{\operatorname{erfi}(\sqrt{5} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{5} \sqrt{\pi}}{160 a^{5}}+\frac{\operatorname{erf}(\sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{16 a^{5}}+\frac{\operatorname{erfi}(\sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{16 a^{5}}$

$$
-\frac{\operatorname{erf}(\sqrt{3} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{3} \sqrt{\pi}}{32 a^{5}}-\frac{\operatorname{erfi}(\sqrt{3} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{3} \sqrt{\pi}}{32 a^{5}}
$$

Result(type 8, 12 leaves):


Problem 28: Unable to integrate problem.

$$
\int \frac{x^{3}}{\operatorname{arcsinh}(a x)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 104 leaves, 12 steps):

$$
-\frac{\operatorname{erf}(\sqrt{2} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{2} \sqrt{\pi}}{4 a^{4}}-\frac{\operatorname{erfi}(\sqrt{2} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{2} \sqrt{\pi}}{4 a^{4}}+\frac{\operatorname{erf}(2 \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{4 a^{4}}+\frac{\operatorname{erfi}(2 \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{4 a^{4}}-\frac{2 x^{3} \sqrt{a^{2} x^{2}+1}}{a \sqrt{\operatorname{arcsinh}(a x)}}
$$

Result(type 8, 12 leaves):

$$
\int \frac{x^{3}}{\operatorname{arcsinh}(a x)^{3 / 2}} \mathrm{~d} x
$$

Problem 29: Unable to integrate problem.

$$
\int \frac{x^{2}}{\operatorname{arcsinh}(a x)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 100 leaves, 12 steps):

$$
\frac{\operatorname{erf}(\sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{4 a^{3}}-\frac{\operatorname{erfi}(\sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{4 a^{3}}-\frac{\operatorname{erf}(\sqrt{3} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{3} \sqrt{\pi}}{4 a^{3}}+\frac{\operatorname{erfi}(\sqrt{3} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{3} \sqrt{\pi}}{4 a^{3}}-\frac{2 x^{2} \sqrt{a^{2} x^{2}+1}}{a \sqrt{\operatorname{arcsinh}(a x)}}
$$

Result(type 8, 12 leaves):

$$
\int \frac{x^{2}}{\operatorname{arcsinh}(a x)^{3 / 2}} \mathrm{~d} x
$$

Problem 31: Unable to integrate problem.

$$
\int \frac{x^{2}}{\operatorname{arcsinh}(a x)^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 170 leaves, 22 steps):

$$
\begin{aligned}
& -\frac{8 x}{15 a^{2} \operatorname{arcsinh}(a x)^{3 / 2}}-\frac{4 x^{3}}{5 \operatorname{arcsinh}(a x)^{3 / 2}}+\frac{\operatorname{erf}(\sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{15 a^{3}}-\frac{\operatorname{erfi}(\sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi}}{15 a^{3}}-\frac{3 \operatorname{erf}(\sqrt{3} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{3} \sqrt{\pi}}{5 a^{3}} \\
& \quad+\frac{3 \operatorname{erfi}(\sqrt{3} \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{3} \sqrt{\pi}}{5 a^{3}}-\frac{2 x^{2} \sqrt{a^{2} x^{2}+1}}{5 a \operatorname{arcsinh}(a x)^{5 / 2}}-\frac{16 \sqrt{a^{2} x^{2}+1}}{15 a^{3} \sqrt{\operatorname{arcsinh}(a x)}}-\frac{24 x^{2} \sqrt{a^{2} x^{2}+1}}{5 a \sqrt{\operatorname{arcsinh}(a x)}}
\end{aligned}
$$

Result(type 8, 12 leaves):

$$
\int \frac{x^{2}}{\operatorname{arcsinh}(a x)^{7 / 2}} \mathrm{~d} x
$$

Problem 34: Unable to integrate problem.

$$
\int x^{m} \operatorname{arcsinh}(a x)^{2} \mathrm{~d} x
$$

Optimal(type 5, 119 leaves, 2 steps):
$\frac{x^{1+m} \operatorname{arcsinh}(a x)^{2}}{1+m}-\frac{2 a x^{2+m} \operatorname{arcsinh}(a x) \text { hypergeom }\left(\left[\frac{1}{2}, 1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-a^{2} x^{2}\right)}{m^{2}+3 m+2}$
$+\frac{2 a^{2} x^{3+m} \text { HypergeometricPFQ }\left(\left[1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right],\left[2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right],-a^{2} x^{2}\right)}{m^{3}+6 m^{2}+11 m+6}$
Result(type 8, 12 leaves):
$\int x^{m} \operatorname{arcsinh}(a x)^{2} \mathrm{~d} x$

Problem 35: Unable to integrate problem.
$\int x^{m} \operatorname{arcsinh}(a x) \mathrm{d} x$
Optimal(type 5, 56 leaves, 2 steps):

$$
\frac{x^{1+m} \operatorname{arcsinh}(a x)}{1+m}-\frac{a x^{2+m} \text { hypergeom }\left(\left[\frac{1}{2}, 1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-a^{2} x^{2}\right)}{m^{2}+3 m+2}
$$

Result(type 8, 10 leaves):

$$
\int x^{m} \operatorname{arcsinh}(a x) \mathrm{d} x
$$

Problem 39: Unable to integrate problem.

$$
\int x^{2} \operatorname{arcsinh}(a x)^{n} \mathrm{~d} x
$$

Optimal(type 4, 105 leaves, 9 steps):

$$
\frac{3^{-1-n} \operatorname{arcsinh}(a x)^{n} \Gamma(1+n,-3 \operatorname{arcsinh}(a x))}{8 a^{3}(-\operatorname{arcsinh}(a x))^{n}}-\frac{\operatorname{arcsinh}(a x)^{n} \Gamma(1+n,-\operatorname{arcsinh}(a x))}{8 a^{3}(-\operatorname{arcsinh}(a x))^{n}}+\frac{\Gamma(1+n, \operatorname{arcsinh}(a x))}{8 a^{3}}-\frac{3^{-1-n} \Gamma(1+n, 3 \operatorname{arcsinh}(a x))}{8 a^{3}}
$$

Result (type 8, 12 leaves):

$$
\int x^{2} \operatorname{arcsinh}(a x)^{n} \mathrm{~d} x
$$

Problem 40: Result unnecessarily involves higher level functions.

$$
\int \operatorname{arcsinh}(a x)^{n} \mathrm{~d} x
$$

Optimal(type 4, 45 leaves, 4 steps):

$$
\frac{\operatorname{arcsinh}(a x)^{n} \Gamma(1+n,-\operatorname{arcsinh}(a x))}{2 a(-\operatorname{arcsinh}(a x))^{n}}-\frac{\Gamma(1+n, \operatorname{arcsinh}(a x))}{2 a}
$$

Result(type 5, 39 leaves):

$$
\frac{\operatorname{arcsinh}(a x)^{1+n} \text { hypergeom }\left(\left[\frac{1}{2}+\frac{n}{2}\right],\left[\frac{1}{2}, \frac{3}{2}+\frac{n}{2}\right], \frac{\operatorname{arcsinh}(a x)^{2}}{4}\right)}{a(1+n)}
$$

Problem 41: Unable to integrate problem.

$$
\int x^{2}(a+b \operatorname{arcsinh}(c x))^{5 / 2} \mathrm{~d} x
$$

Optimal(type 4, 256 leaves, 24 steps):
$\frac{x^{3}(a+b \operatorname{arcsinh}(c x))^{5 / 2}}{3}+\frac{5 b^{5 / 2} \mathrm{e}^{\frac{3 a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{1728 c^{3}}-\frac{5 b^{5 / 2} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{1728 c^{3} \mathrm{e}^{\frac{3 a}{b}}}$

$-\frac{5 b x^{2}(a+b \operatorname{arcsinh}(c x))^{3 / 2} \sqrt{c^{2} x^{2}+1}}{18 c}-\frac{5 b^{2} x \sqrt{a+b \operatorname{arcsinh}(c x)}}{6 c^{2}}+\frac{5 b^{2} x^{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{36}$
Result(type 8, 16 leaves):

$$
\int x^{2}(a+b \operatorname{arcsinh}(c x))^{5 / 2} \mathrm{~d} x
$$



Optimal(type 4, 67 leaves, 6 steps):


Result(type 8, 12 leaves):

$$
\int \frac{1}{\sqrt{a+b \operatorname{arcsinh}(c x)}} \mathrm{d} x
$$

Problem 43: Unable to integrate problem.


Optimal(type 4, 179 leaves, 12 steps):


$$
+\frac{\operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{4 b^{3 / 2} c^{3} \mathrm{e}^{\frac{3 a}{b}}}-\frac{2 x^{2} \sqrt{c^{2} x^{2}+1}}{b c \sqrt{a+b \operatorname{arcsinh}(c x)}}
$$

Result(type 8, 16 leaves):


Problem 44: Unable to integrate problem.

$$
\int \frac{x}{(a+b \operatorname{arcsinh}(c x))^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 147 leaves, 11 steps):
$-\frac{2 \mathrm{e}^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{3 b^{5 / 2} c^{2}}+\frac{2 \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{3 b^{5 / 2} c^{2} \mathrm{e}^{\frac{2 a}{b}}}-\frac{2 x \sqrt{c^{2} x^{2}+1}}{3 b c(a+b \operatorname{arcsinh}(c x))^{3 / 2}}$

$$
-\frac{4}{3 b^{2} c^{2} \sqrt{a+b \operatorname{arcsinh}(c x)}}-\frac{8 x^{2}}{3 b^{2} \sqrt{a+b \operatorname{arcsinh}(c x)}}
$$

Result(type 8, 14 leaves):

$$
\int \frac{x}{(a+b \operatorname{arcsinh}(c x))^{5 / 2}} \mathrm{~d} x
$$

Problem 45: Unable to integrate problem.

$$
\int \frac{x}{(a+b \operatorname{arcsinh}(c x))^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 177 leaves, 9 steps):
$-\frac{4}{15 b^{2} c^{2}(a+b \operatorname{arcsinh}(c x))^{3 / 2}}-\frac{8 x^{2}}{15 b^{2}(a+b \operatorname{arcsinh}(c x))^{3 / 2}}+\frac{8 \mathrm{e}^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{15 b^{7 / 2} c^{2}}$

$$
+\frac{8 \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{15 b^{7 / 2} c^{2} \mathrm{e}^{\frac{2 a}{b}}}-\frac{2 x \sqrt{c^{2} x^{2}+1}}{5 b c(a+b \operatorname{arcsinh}(c x))^{5 / 2}}-\frac{32 x \sqrt{c^{2} x^{2}+1}}{15 b^{3} c \sqrt{a+b \operatorname{arcsinh}(c x)}}
$$

Result(type 8, 14 leaves):

$$
\int \frac{x}{(a+b \operatorname{arcsinh}(c x))^{7 / 2}} \mathrm{~d} x
$$

Problem 46: Unable to integrate problem.

$$
\int \frac{1}{(a+b \operatorname{arcsinh}(c x))^{7 / 2}} d x
$$

Optimal(type 4, 141 leaves, 9 steps):
$-\frac{4 x}{15 b^{2}(a+b \operatorname{arcsinh}(c x))^{3 / 2}}-\frac{4 \mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{15 b^{7 / 2} c}+\frac{4 \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{15 b^{7 / 2} c \mathrm{e}^{\frac{a}{b}}}-\frac{2 \sqrt{c^{2} x^{2}+1}}{5 b c(a+b \operatorname{arcsinh}(c x))^{5 / 2}}$
$-\frac{8 \sqrt{c^{2} x^{2}+1}}{15 b^{3} c \sqrt{a+b \operatorname{arcsinh}(c x)}}$
Result(type 8, 12 leaves):

$$
\int \frac{1}{(a+b \operatorname{arcsinh}(c x))^{7 / 2}} \mathrm{~d} x
$$

Test results for the 178 problems in "7.1.4 (f $x)^{\wedge} m\left(d+e x^{\wedge} 2\right)^{\wedge} p(a+b$ arcsinh(c $\left.x)\right)^{\wedge} n . t x t "$
Problem 21: Result more than twice size of optimal antiderivative.

$$
\int x\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x)) \mathrm{d} x
$$

Optimal(type 3, 61 leaves, 3 steps):

$$
-\frac{b \pi^{3 / 2} x}{5 c}-\frac{2 b c \pi^{3 / 2} x^{3}}{15}-\frac{b c^{3} \pi^{3 / 2} x^{5}}{25}+\frac{\left(c^{2} \pi x^{2}+\pi\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))}{5 c^{2} \pi}
$$

Result(type 3, 138 leaves):
$\frac{a\left(c^{2} \pi x^{2}+\pi\right)^{5 / 2}}{5 c^{2} \pi}$
$+\frac{1}{75 c^{2} \sqrt{c^{2} x^{2}+1}}\left(b \pi^{3 / 2}\left(15 \operatorname{arcsinh}(c x) x^{6} c^{6}+45 \operatorname{arcsinh}(c x) c^{4} x^{4}-3 \sqrt{c^{2} x^{2}+1} x^{5} c^{5}+45 \operatorname{arcsinh}(c x) c^{2} x^{2}-10 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}\right.\right.$
$\left.\left.+15 \operatorname{arcsinh}(c x)-15 \sqrt{c^{2} x^{2}+1} c x\right)\right)$

Problem 23: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{x^{4}} \mathrm{~d} x
$$

Optimal(type 3, 97 leaves, 6 steps):

$$
-\frac{b c \pi^{3 / 2}}{6 x^{2}}-\frac{\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3 x^{3}}+\frac{c^{3} \pi^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{2 b}+\frac{4 b c^{3} \pi^{3 / 2} \ln (x)}{3}-\frac{c^{2} \pi(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} \pi x^{2}+\pi}}{x}
$$

Result(type 3, 621 leaves):

$$
\begin{array}{r}
-\frac{a\left(c^{2} \pi x^{2}+\pi\right)^{5 / 2}}{3 \pi x^{3}}-\frac{2 a c^{2}\left(c^{2} \pi x^{2}+\pi\right)^{5 / 2}}{3 \pi x}+\frac{2 a c^{4} x\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}}{3}+a c^{4} \sqrt{c^{2} \pi x^{2}+\pi} \pi x+\frac{a c^{4} \pi^{2} \ln \left(\frac{\pi x c^{2}}{\sqrt{c^{2} \pi}}+\sqrt{c^{2} \pi x^{2}+\pi}\right)}{\sqrt{c^{2} \pi}} \\
\quad+\frac{b c^{3} \pi^{3 / 2} \operatorname{arcsinh}(c x)^{2}}{2}-\frac{8 b \pi^{3 / 2} c^{3} \operatorname{arcsinh}(c x)}{3}+\frac{32 b \pi^{3 / 2} x^{4} \operatorname{arcsinh}(c x) c^{7}}{24 c^{4} x^{4}+9 c^{2} x^{2}+1}-\frac{32 b \pi^{3 / 2} x^{3} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) c^{6}}{24 c^{4} x^{4}+9 c^{2} x^{2}+1}
\end{array}
$$

$$
\begin{aligned}
& +\frac{8 b \pi^{3 / 2} x^{4} c^{7}}{3\left(24 c^{4} x^{4}+9 c^{2} x^{2}+1\right)}-\frac{8 b \pi^{3 / 2} x^{2}\left(c^{2} x^{2}+1\right) c^{5}}{3\left(24 c^{4} x^{4}+9 c^{2} x^{2}+1\right)}+\frac{12 b \pi^{3 / 2} x^{2} \operatorname{arcsinh}(c x) c^{5}}{24 c^{4} x^{4}+9 c^{2} x^{2}+1}-\frac{20 b \pi^{3 / 2} x \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) c^{4}}{24 c^{4} x^{4}+9 c^{2} x^{2}+1} \\
& -\frac{4 b \pi^{3 / 2}\left(c^{2} x^{2}+1\right) c^{3}}{3\left(24 c^{4} x^{4}+9 c^{2} x^{2}+1\right)}+\frac{4 b \pi^{3 / 2} \operatorname{arcsinh}(c x) c^{3}}{3\left(24 c^{4} x^{4}+9 c^{2} x^{2}+1\right)}-\frac{13 b \pi^{3 / 2} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) c^{2}}{3\left(24 c^{4} x^{4}+9 c^{2} x^{2}+1\right) x}-\frac{b \pi^{3 / 2}\left(c^{2} x^{2}+1\right) c}{6\left(24 c^{4} x^{4}+9 c^{2} x^{2}+1\right) x^{2}} \\
& -\frac{b \pi^{3 / 2} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)}{3\left(24 c^{4} x^{4}+9 c^{2} x^{2}+1\right) x^{3}}+\frac{4 b c^{3} \pi^{3 / 2} \ln \left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}-1\right)}{3}
\end{aligned}
$$

Problem 26: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}(a+b \operatorname{arcsinh}(c x))}{\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 113 leaves, 7 steps):

$$
-\frac{b x^{2}}{4 c^{3} \pi^{3 / 2}}-\frac{3(a+b \operatorname{arcsinh}(c x))^{2}}{4 b c^{5} \pi^{3 / 2}}-\frac{b \ln \left(c^{2} x^{2}+1\right)}{2 c^{5} \pi^{3 / 2}}-\frac{x^{3}(a+b \operatorname{arcsinh}(c x))}{c^{2} \pi \sqrt{c^{2} \pi x^{2}+\pi}}+\frac{3 x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} \pi x^{2}+\pi}}{2 c^{4} \pi^{2}}
$$

Result(type 3, 268 leaves):

$$
\begin{aligned}
& \frac{a x^{3}}{2 c^{2} \pi \sqrt{c^{2} \pi x^{2}+\pi}}+\frac{3 a x}{2 c^{4} \pi \sqrt{c^{2} \pi x^{2}+\pi}}-\frac{3 a \ln \left(\frac{\pi x c^{2}}{\sqrt{c^{2} \pi}}+\sqrt{c^{2} \pi x^{2}+\pi}\right)}{2 c^{4} \pi \sqrt{c^{2} \pi}}-\frac{3 b \operatorname{arcsinh}(c x)^{2}}{4 c^{5} \pi^{3 / 2}}+\frac{b x \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1}}{2 \pi^{3 / 2} c^{4}}-\frac{b x^{2}}{4 c^{3} \pi^{3 / 2}} \\
& \quad+\frac{2 b \operatorname{arcsinh}(c x)}{c^{5} \pi^{3 / 2}}-\frac{b}{8 \pi^{3 / 2} c^{5}}-\frac{b \operatorname{arcsinh}(c x) x^{2}}{\pi^{3 / 2} c^{3}\left(c^{2} x^{2}+1\right)}+\frac{b \operatorname{arcsinh}(c x) x}{\pi^{3 / 2} c^{4} \sqrt{c^{2} x^{2}+1}}-\frac{b \operatorname{arcsinh}(c x)}{\pi^{3 / 2} c^{5}\left(c^{2} x^{2}+1\right)}-\frac{b \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{c^{5} \pi^{3 / 2}}
\end{aligned}
$$

Problem 27: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}(a+b \operatorname{arcsinh}(c x))}{\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 78 leaves, 4 steps):

$$
-\frac{b x}{c^{3} \pi^{3 / 2}}-\frac{b \arctan (c x)}{c^{4} \pi^{3 / 2}}+\frac{a+b \operatorname{arcsinh}(c x)}{c^{4} \pi \sqrt{c^{2} \pi x^{2}+\pi}}+\frac{(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} \pi x^{2}+\pi}}{c^{4} \pi^{2}}
$$

Result(type 3, 157 leaves):
$\frac{a x^{2}}{c^{2} \pi \sqrt{c^{2} \pi x^{2}+\pi}}+\frac{2 a}{\pi c^{4} \sqrt{c^{2} \pi x^{2}+\pi}}+\frac{b \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)}{\pi^{3 / 2} c^{4}}-\frac{b x}{c^{3} \pi^{3 / 2}}+\frac{b \operatorname{arcsinh}(c x)}{\pi^{3 / 2} \sqrt{c^{2} x^{2}+1} c^{4}}+\frac{\mathrm{I} b \ln \left(c x+\sqrt{c^{2} x^{2}+1}-\mathrm{I}\right)}{c^{4} \pi^{3 / 2}}$
$-\frac{\mathrm{I} b \ln \left(c x+\sqrt{c^{2} x^{2}+1}+\mathrm{I}\right)}{c^{4} \pi^{3 / 2}}$

Problem 28: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{6}(a+b \operatorname{arcsinh}(c x))}{\left(c^{2} \pi x^{2}+\pi\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 164 leaves, 11 steps):

$$
\begin{aligned}
& -\frac{b x^{2}}{4 c^{5} \pi^{5 / 2}}-\frac{b}{6 c^{7} \pi^{5 / 2}\left(c^{2} x^{2}+1\right)}-\frac{x^{5}(a+b \operatorname{arcsinh}(c x))}{3 c^{2} \pi\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}}-\frac{5(a+b \operatorname{arcsinh}(c x))^{2}}{4 b c^{7} \pi^{5 / 2}}-\frac{7 b \ln \left(c^{2} x^{2}+1\right)}{6 c^{7} \pi^{5 / 2}}-\frac{5 x^{3}(a+b \operatorname{arcsinh}(c x))}{3 c^{4} \pi^{2} \sqrt{c^{2} \pi x^{2}+\pi}} \\
& \quad+\frac{5 x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} \pi x^{2}+\pi}}{2 c^{6} \pi^{3}}
\end{aligned}
$$

Result(type 3, 969 leaves):

$$
\begin{aligned}
& \frac{5 a x^{3}}{6 c^{4} \pi\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}}+\frac{5 a x}{2 c^{6} \pi^{2} \sqrt{c^{2} \pi x^{2}+\pi}}-\frac{5 a \ln \left(\frac{\pi x c^{2}}{\sqrt{c^{2} \pi}}+\sqrt{c^{2} \pi x^{2}+\pi}\right)}{2 c^{6} \pi^{2} \sqrt{c^{2} \pi}}+\frac{a x^{5}}{2 c^{2} \pi\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}} \\
& -\frac{49 b}{6 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2} c^{7}}-\frac{b}{8 \pi^{5 / 2} c^{7}}-\frac{7 b \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 c^{7} \pi^{5 / 2}}+\frac{14 b \operatorname{arcsinh}(c x)}{3 c^{7} \pi^{5 / 2}}-\frac{5 b \operatorname{arcsinh}(c x)^{2}}{4 c^{7} \pi^{5 / 2}} \\
& -\frac{b x^{2}}{4 c^{5} \pi^{5 / 2}}+\frac{385 b \operatorname{arcsinh}(c x) x^{5}}{\pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{3 / 2} c^{2}}+\frac{1009 b \operatorname{arcsinh}(c x) x^{3}}{3 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{3 / 2} c^{4}} \\
& +\frac{98 b x \operatorname{arcsinh}(c x)}{\pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{3 / 2} c^{6}}-\frac{1463 b \operatorname{arcsinh}(c x) x^{2}}{3 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2} c^{5}} \\
& -\frac{147 b c \operatorname{arcsinh}(c x) x^{8}}{\pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2}}-\frac{553 b \operatorname{arcsinh}(c x) x^{6}}{\pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2} c}-\frac{2338 b \operatorname{arcsinh}(c x) x^{4}}{3 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2} c^{3}} \\
& +\frac{147 b \operatorname{arcsinh}(c x) x^{7}}{\pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{6 b x^{2}}{\pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right) c^{5}}+\frac{49 b x^{6}}{6 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right) c} \\
& +\frac{14 b x^{4}}{\pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right) c^{3}}+\frac{b x \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1}}{2 \pi^{5 / 2} c^{6}}-\frac{49 b c x^{8}}{6 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2}} \\
& -\frac{98 b x^{6}}{3 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2} c}-\frac{49 b x^{4}}{\pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2} c^{3}} \\
& -\frac{98 b x^{2}}{3 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2} c^{5}}-\frac{343 b \operatorname{arcsinh}(c x)}{3 \pi^{5 / 2}\left(63 c^{4} x^{4}+111 c^{2} x^{2}+49\right)\left(c^{2} x^{2}+1\right)^{2} c^{7}}
\end{aligned}
$$

Problem 30: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}(a+b \operatorname{arcsinh}(c x))}{\left(c^{2} \pi x^{2}+\pi\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 121 leaves, 7 steps):

$$
\frac{b}{6 c^{5} \pi^{5 / 2}\left(c^{2} x^{2}+1\right)}-\frac{x^{3}(a+b \operatorname{arcsinh}(c x))}{3 c^{2} \pi\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}}+\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{2 b c^{5} \pi^{5 / 2}}+\frac{2 b \ln \left(c^{2} x^{2}+1\right)}{3 c^{5} \pi^{5 / 2}}-\frac{x(a+b \operatorname{arcsinh}(c x))}{c^{4} \pi^{2} \sqrt{c^{2} \pi x^{2}+\pi}}
$$

Result(type 3, 896 leaves):

$$
\begin{aligned}
& -\frac{a x^{3}}{3 c^{2} \pi\left(c^{2} \pi x^{2}+\pi\right)^{3 / 2}}-\frac{a x}{c^{4} \pi^{2} \sqrt{c^{2} \pi x^{2}+\pi}}+\frac{a \ln \left(\frac{\pi x c^{2}}{\sqrt{c^{2} \pi}}+\sqrt{c^{2} \pi x^{2}+\pi}\right)}{c^{4} \pi^{2} \sqrt{c^{2} \pi}}+\frac{b \operatorname{arcsinh}(c x)^{2}}{2 c^{5} \pi^{5 / 2}}-\frac{8 b \operatorname{arcsinh}(c x)}{3 c^{5} \pi^{5 / 2}} \\
& +\frac{32 b c^{3} \operatorname{arcsinh}(c x) x^{8}}{\pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right)\left(c^{2} x^{2}+1\right)^{2}}-\frac{32 b c^{2} \operatorname{arcsinh}(c x) x^{7}}{\pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right)\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{8 b c^{3} x^{8}}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right)\left(c^{2} x^{2}+1\right)^{2}} \\
& -\frac{8 b c x^{6}}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right)\left(c^{2} x^{2}+1\right)}+\frac{116 b c \operatorname{arcsinh}(c x) x^{6}}{\pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right)\left(c^{2} x^{2}+1\right)^{2}}-\frac{76 b \operatorname{arcsinh}(c x) x^{5}}{\pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right)\left(c^{2} x^{2}+1\right)^{3 / 2}} \\
& +\frac{32 b c x^{6}}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right)\left(c^{2} x^{2}+1\right)^{2}}-\frac{4 b x^{4}}{\pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c\left(c^{2} x^{2}+1\right)}+\frac{472 b \operatorname{arcsinh}(c x) x^{4}}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c\left(c^{2} x^{2}+1\right)^{2}} \\
& -\frac{181 b \operatorname{arcsinh}(c x) x^{3}}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c^{2}\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{16 b x^{4}}{\pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c\left(c^{2} x^{2}+1\right)^{2}}-\frac{3 b x^{2}}{2 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c^{3}\left(c^{2} x^{2}+1\right)} \\
& +\frac{284 b \operatorname{arcsinh}(c x) x^{2}}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c^{3}\left(c^{2} x^{2}+1\right)^{2}}-\frac{16 b x \operatorname{arcsinh}(c x)}{\pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c^{4}\left(c^{2} x^{2}+1\right)^{3 / 2}} \\
& +\frac{32 b x^{2}}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c^{3}\left(c^{2} x^{2}+1\right)^{2}}+\frac{64 b \operatorname{arcsinh}(c x)}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c^{5}\left(c^{2} x^{2}+1\right)^{2}} \\
& +\frac{8 b}{3 \pi^{5 / 2}\left(24 c^{4} x^{4}+39 c^{2} x^{2}+16\right) c^{5}\left(c^{2} x^{2}+1\right)^{2}}+\frac{4 b \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 c^{5} \pi^{5 / 2}}
\end{aligned}
$$

Problem 33: Result more than twice size of optimal antiderivative.

$$
\int \frac{\operatorname{arcsinh}(a x)}{x^{2} \sqrt{a^{2} x^{2}+1}} d x
$$

Optimal(type 3, 25 leaves, 2 steps):

$$
a \ln (x)-\frac{\operatorname{arcsinh}(a x) \sqrt{a^{2} x^{2}+1}}{x}
$$

Result(type 3, 55 leaves):

$$
-2 a \operatorname{arcsinh}(a x)+\frac{\left(a x-\sqrt{a^{2} x^{2}+1}\right) \operatorname{arcsinh}(a x)}{x}+a \ln \left(\left(a x+\sqrt{a^{2} x^{2}+1}\right)^{2}-1\right)
$$

Problem 35: Result more than twice size of optimal antiderivative.

$$
\int x^{3}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d} \mathrm{~d} x
$$

Optimal(type 3, 149 leaves, 3 steps):

$$
-\frac{\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3 c^{4} d}+\frac{\left(c^{2} d x^{2}+d\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))}{5 c^{4} d^{2}}+\frac{2 b x \sqrt{c^{2} d x^{2}+d}}{15 c^{3} \sqrt{c^{2} x^{2}+1}}-\frac{b x^{3} \sqrt{c^{2} d x^{2}+d}}{45 c \sqrt{c^{2} x^{2}+1}}-\frac{b c x^{5} \sqrt{c^{2} d x^{2}+d}}{25 \sqrt{c^{2} x^{2}+1}}
$$

Result(type 3, 577 leaves):
$a\left(\frac{x^{2}\left(c^{2} d x^{2}+d\right)^{3 / 2}}{5 c^{2} d}-\frac{2\left(c^{2} d x^{2}+d\right)^{3 / 2}}{15 d c^{4}}\right)$
$+b\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(16 x^{6} c^{6}+16 \sqrt{c^{2} x^{2}+1} x^{5} c^{5}+28 c^{4} x^{4}+20 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+13 c^{2} x^{2}+5 \sqrt{c^{2} x^{2}+1} c x+1\right)(-1+5 \operatorname{arcsinh}(c x))}{800 c^{4}\left(c^{2} x^{2}+1\right)}\right.$
$-\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}+4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}+3 \sqrt{c^{2} x^{2}+1} c x+1\right)(-1+3 \operatorname{arcsinh}(c x))}{288 c^{4}\left(c^{2} x^{2}+1\right)}$
$-\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+\sqrt{c^{2} x^{2}+1} c x+1\right)(-1+\operatorname{arcsinh}(c x))}{16 c^{4}\left(c^{2} x^{2}+1\right)}-\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}-\sqrt{c^{2} x^{2}+1} c x+1\right)(1+\operatorname{arcsinh}(c x))}{16 c^{4}\left(c^{2} x^{2}+1\right)}$
$-\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}-4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}-3 \sqrt{c^{2} x^{2}+1} c x+1\right)(1+3 \operatorname{arcsinh}(c x))}{288 c^{4}\left(c^{2} x^{2}+1\right)}$
$\left.+\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(16 x^{6} c^{6}-16 \sqrt{c^{2} x^{2}+1} x^{5} c^{5}+28 c^{4} x^{4}-20 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+13 c^{2} x^{2}-5 \sqrt{c^{2} x^{2}+1} c x+1\right)(1+5 \operatorname{arcsinh}(c x))}{800 c^{4}\left(c^{2} x^{2}+1\right)}\right)$

Problem 36: Result more than twice size of optimal antiderivative.

$$
\int x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d} \mathrm{~d} x
$$

Optimal(type 3, 89 leaves, 2 steps):

$$
\frac{\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3 c^{2} d}-\frac{b x \sqrt{c^{2} d x^{2}+d}}{3 c \sqrt{c^{2} x^{2}+1}}-\frac{b c x^{3} \sqrt{c^{2} d x^{2}+d}}{9 \sqrt{c^{2} x^{2}+1}}
$$

Result(type 3, 320 leaves):
$\frac{a\left(c^{2} d x^{2}+d\right)^{3 / 2}}{3 c^{2} d}+b\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}+4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}+3 \sqrt{c^{2} x^{2}+1} c x+1\right)(-1+3 \operatorname{arcsinh}(c x))}{72\left(c^{2} x^{2}+1\right) c^{2}}\right.$

$$
\begin{aligned}
& +\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+\sqrt{c^{2} x^{2}+1} c x+1\right)(-1+\operatorname{arcsinh}(c x))}{8\left(c^{2} x^{2}+1\right) c^{2}}+\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}-\sqrt{c^{2} x^{2}+1} c x+1\right)(1+\operatorname{arcsinh}(c x))}{8\left(c^{2} x^{2}+1\right) c^{2}} \\
& \left.+\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}-4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}-3 \sqrt{c^{2} x^{2}+1} c x+1\right)(1+3 \operatorname{arcsinh}(c x))}{72\left(c^{2} x^{2}+1\right) c^{2}}\right)
\end{aligned}
$$

Problem 39: Result more than twice size of optimal antiderivative.

$$
\int\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x)) \mathrm{d} x
$$

Optimal(type 3, 154 leaves, 6 steps):
$\frac{x\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{4}+\frac{3 d x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{8}-\frac{5 b c d x^{2} \sqrt{c^{2} d x^{2}+d}}{16 \sqrt{c^{2} x^{2}+1}}-\frac{b c^{3} d x^{4} \sqrt{c^{2} d x^{2}+d}}{16 \sqrt{c^{2} x^{2}+1}}$

$$
+\frac{3 d(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{16 b c \sqrt{c^{2} x^{2}+1}}
$$

Result(type 3, 317 leaves):
$\frac{a x\left(c^{2} d x^{2}+d\right)^{3 / 2}}{4}+\frac{3 a d x \sqrt{c^{2} d x^{2}+d}}{8}+\frac{3 a d^{2} \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{8 \sqrt{c^{2} d}}+\frac{3 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} d}{16 \sqrt{c^{2} x^{2}+1} c}-\frac{17 b \sqrt{d\left(c^{2} x^{2}+1\right)} d}{128 c \sqrt{c^{2} x^{2}+1}}$
$+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{4} \operatorname{arcsinh}(c x) x^{5}}{4\left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{3} x^{4}}{16 \sqrt{c^{2} x^{2}+1}}+\frac{7 b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{2} \operatorname{arcsinh}(c x) x^{3}}{8\left(c^{2} x^{2}+1\right)}-\frac{5 b \sqrt{d\left(c^{2} x^{2}+1\right)} d c x^{2}}{16 \sqrt{c^{2} x^{2}+1}}$
$+\frac{5 b \sqrt{d\left(c^{2} x^{2}+1\right)} d x \operatorname{arcsinh}(c x)}{8\left(c^{2} x^{2}+1\right)}$

Problem 40: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 155 leaves, 6 steps):
$-\frac{\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{x}+\frac{3 c^{2} d x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{2}-\frac{b c^{3} d x^{2} \sqrt{c^{2} d x^{2}+d}}{4 \sqrt{c^{2} x^{2}+1}}+\frac{3 c d(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{4 b \sqrt{c^{2} x^{2}+1}}$

$$
+\frac{b c d \ln (x) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}}
$$

Result(type 3, 391 leaves):

$$
\begin{aligned}
& -\frac{a\left(c^{2} d x^{2}+d\right)^{5 / 2}}{d x}+a c^{2} x\left(c^{2} d x^{2}+d\right)^{3 / 2}+\frac{3 a \sqrt{c^{2} d x^{2}+d} x c^{2} d}{2}+\frac{3 a c^{2} d^{2} \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{2 \sqrt{c^{2} d}}+\frac{3 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} d c}{4 \sqrt{c^{2} x^{2}+1}} \\
& \quad+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{4} \operatorname{arcsinh}(c x) x^{3}}{2\left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{3} x^{2}}{4 \sqrt{c^{2} x^{2}+1}}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{2} x \operatorname{arcsinh}(c x)}{2\left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} d c \operatorname{arcsinh}(c x)}{\sqrt{c^{2} x^{2}+1}} \\
& -\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} d c}{8 \sqrt{c^{2} x^{2}+1}}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) d}{x\left(c^{2} x^{2}+1\right)}+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \ln \left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}-1\right) d c}{\sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Problem 42: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}(a+b \operatorname{arcsinh}(c x))}{\sqrt{c^{2} d x^{2}+d}} \mathrm{~d} x
$$

Optimal(type 3, 122 leaves, 4 steps):

$$
\frac{2 b x \sqrt{c^{2} x^{2}+1}}{3 c^{3} \sqrt{c^{2} d x^{2}+d}}-\frac{b x^{3} \sqrt{c^{2} x^{2}+1}}{9 c \sqrt{c^{2} d x^{2}+d}}-\frac{2(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{3 c^{4} d}+\frac{x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{3 c^{2} d}
$$

Result(type 3, 357 leaves):

$$
\begin{aligned}
& a\left(\frac{x^{2} \sqrt{c^{2} d x^{2}+d}}{3 c^{2} d}-\frac{2 \sqrt{c^{2} d x^{2}+d}}{3 d c^{4}}\right)+b\left(\frac{(-1+3 \operatorname{arcsinh}(c x)) \sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}+4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}+3 \sqrt{c^{2} x^{2}+1} c x+1\right)}{72 c^{4} d\left(c^{2} x^{2}+1\right)}\right. \\
& \quad-\frac{3(-1+\operatorname{arcsinh}(c x)) \sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+\sqrt{c^{2} x^{2}+1} c x+1\right)}{8 c^{4} d\left(c^{2} x^{2}+1\right)}-\frac{3(1+\operatorname{arcsinh}(c x)) \sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}-\sqrt{c^{2} x^{2}+1} c x+1\right)}{8 c^{4} d\left(c^{2} x^{2}+1\right)} \\
& \left.\quad+\frac{(1+3 \operatorname{arcsinh}(c x)) \sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}-4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}-3 \sqrt{c^{2} x^{2}+1} c x+1\right)}{72 c^{4} d\left(c^{2} x^{2}+1\right)}\right)
\end{aligned}
$$

Problem 43: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}(a+b \operatorname{arcsinh}(c x))}{\sqrt{c^{2} d x^{2}+d}} \mathrm{~d} x
$$

Optimal(type 3, 103 leaves, 3 steps):

$$
-\frac{b x^{2} \sqrt{c^{2} x^{2}+1}}{4 c \sqrt{c^{2} d x^{2}+d}}-\frac{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}}{4 b c^{3} \sqrt{c^{2} d x^{2}+d}}+\frac{x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{2 c^{2} d}
$$

Result(type 3, 246 leaves):

$$
\begin{aligned}
& \frac{a x \sqrt{c^{2} d x^{2}+d}}{2 c^{2} d}-\frac{a \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{2 c^{2} \sqrt{c^{2} d}}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2}}{4 \sqrt{c^{2} x^{2}+1} c^{3} d}+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{3}}{2 d\left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{2}}{4 c d \sqrt{c^{2} x^{2}+1}} \\
& \quad+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} x \operatorname{arcsinh}(c x)}{2 c^{2} d\left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)}}{8 c^{3} d \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Problem 44: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{x^{2} \sqrt{c^{2} d x^{2}+d}} \mathrm{~d} x
$$

Optimal(type 3, 57 leaves, 2 steps):

$$
\frac{b c \ln (x) \sqrt{c^{2} x^{2}+1}}{\sqrt{c^{2} d x^{2}+d}}-\frac{(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{d x}
$$

Result(type 3, 182 leaves):

$$
\begin{aligned}
& -\frac{a \sqrt{c^{2} d x^{2}+d}}{d x}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) c}{\sqrt{c^{2} x^{2}+1} d}-\frac{b \operatorname{arcsinh}(c x) \sqrt{d\left(c^{2} x^{2}+1\right)} x c^{2}}{\left(c^{2} x^{2}+1\right) d}-\frac{b \operatorname{arcsinh}(c x) \sqrt{d\left(c^{2} x^{2}+1\right)}}{\left(c^{2} x^{2}+1\right) x d} \\
& +\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \ln \left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}-1\right) c}{\sqrt{c^{2} x^{2}+1} d}
\end{aligned}
$$

Problem 46: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}(a+b \operatorname{arcsinh}(c x))}{\left(c^{2} d x^{2}+d\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 182 leaves, 7 steps):
$-\frac{x^{3}(a+b \operatorname{arcsinh}(c x))}{c^{2} d \sqrt{c^{2} d x^{2}+d}}-\frac{b x^{2} \sqrt{c^{2} x^{2}+1}}{4 c^{3} d \sqrt{c^{2} d x^{2}+d}}-\frac{3(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}}{4 b c^{5} d \sqrt{c^{2} d x^{2}+d}}-\frac{b \ln \left(c^{2} x^{2}+1\right) \sqrt{c^{2} x^{2}+1}}{2 c^{5} d \sqrt{c^{2} d x^{2}+d}}+\frac{3 x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{2 c^{4} d^{2}}$
Result(type 3, 365 leaves):

$$
\begin{aligned}
& \frac{a x^{3}}{2 c^{2} d \sqrt{c^{2} d x^{2}+d}}+\frac{3 a x}{2 c^{4} d \sqrt{c^{2} d x^{2}+d}}-\frac{3 a \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{2 c^{4} d \sqrt{c^{2} d}}-\frac{3 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2}}{4 \sqrt{c^{2} x^{2}+1} c^{5} d^{2}}+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{3}}{2 c^{2} d^{2}\left(c^{2} x^{2}+1\right)} \\
& -\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{2}}{4 c^{3} d^{2} \sqrt{c^{2} x^{2}+1}}+\frac{3 b \sqrt{d\left(c^{2} x^{2}+1\right)} x \operatorname{arcsinh(cx)}}{2 c^{4} d^{2}\left(c^{2} x^{2}+1\right)}+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)}{c^{5} d^{2} \sqrt{c^{2} x^{2}+1}}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)}}{8 c^{5} d^{2} \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

```
\(-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{\sqrt{c} x^{2}+1}\)
    \(\sqrt{c^{2} x^{2}+1} c^{5} d^{2}\)
```

Problem 47: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{6}(a+b \operatorname{arcsinh}(c x))}{\left(c^{2} d x^{2}+d\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 245 leaves, 11 steps):

$$
\begin{aligned}
& -\frac{x^{5}(a+b \operatorname{arcsinh}(c x))}{3 c^{2} d\left(c^{2} d x^{2}+d\right)^{3 / 2}}-\frac{5 x^{3}(a+b \operatorname{arcsinh}(c x))}{3 c^{4} d^{2} \sqrt{c^{2} d x^{2}+d}}-\frac{b}{6 c^{7} d^{2} \sqrt{c^{2} x^{2}+1} \sqrt{c^{2} d x^{2}+d}}-\frac{b x^{2} \sqrt{c^{2} x^{2}+1}}{4 c^{5} d^{2} \sqrt{c^{2} d x^{2}+d}}-\frac{5(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}}{4 b c^{7} d^{2} \sqrt{c^{2} d x^{2}+d}} \\
& \quad-\frac{7 b \ln \left(c^{2} x^{2}+1\right) \sqrt{c^{2} x^{2}+1}}{6 c^{7} d^{2} \sqrt{c^{2} d x^{2}+d}}+\frac{5 x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{2 c^{6} d^{3}}
\end{aligned}
$$

Result(type 3, 1606 leaves):

$$
-\frac{147 b \sqrt{d\left(c^{2} x^{2}+1\right)} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) x^{6}}{\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c d^{3}}-\frac{1120 b \sqrt{d\left(c^{2} x^{2}+1\right)} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) x^{2}}{3\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{5} d^{3}}
$$

$$
-\frac{406 b \sqrt{d\left(c^{2} x^{2}+1\right)} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) x^{4}}{\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{3} d^{3}}-\frac{5 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2}}{4 \sqrt{c^{2} x^{2}+1} c^{7} d^{3}}+\frac{14 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)}{3 \sqrt{c^{2} x^{2}+1} c^{7} d^{3}}
$$

$$
+\frac{147 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{7}}{\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) d^{3}}-\frac{7 b \sqrt{d\left(c^{2} x^{2}+1\right)} \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 \sqrt{c^{2} x^{2}+1} c^{7} d^{3}}
$$

$$
+\frac{70 b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{5}}{3\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{2} d^{3}}+\frac{133 b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{3}}{6\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{4} d^{3}}
$$

$$
+\frac{7 b \sqrt{d\left(c^{2} x^{2}+1\right)} x}{\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{6} d^{3}}-\frac{49 b \sqrt{d\left(c^{2} x^{2}+1\right)} \sqrt{c^{2} x^{2}+1}}{6\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{7} d^{3}}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{2}}{4 c^{5} d^{3} \sqrt{c^{2} x^{2}+1}}
$$

$$
+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{3}}{2 c^{4} d^{3}\left(c^{2} x^{2}+1\right)}+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} x \operatorname{arcsinh}(c x)}{2 c^{6} d^{3}\left(c^{2} x^{2}+1\right)}-\frac{49 b \sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+1\right) x^{5}}{6\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{2} d^{3}}
$$

$$
+\frac{385 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{5}}{\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{2} d^{3}}-\frac{21 b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{4} \sqrt{c^{2} x^{2}+1}}{2\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{3} d^{3}}
$$

$$
-\frac{91 b \sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+1\right) x^{3}}{6\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{4} d^{3}}+\frac{1009 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{3}}{3\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{4} d^{3}}
$$

$$
-\frac{37 b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{2} \sqrt{c^{2} x^{2}+1}}{2\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{5} d^{3}}-\frac{7 b \sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+1\right) x}{\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{6} d^{3}}
$$

$$
\begin{aligned}
& +\frac{98 b \sqrt{d\left(c^{2} x^{2}+1\right)} x \operatorname{arcsinh}(c x)}{\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{6} d^{3}}-\frac{343 b \sqrt{d\left(c^{2} x^{2}+1\right)} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)}{3\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) c^{7} d^{3}} \\
& +\frac{49 b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{7}}{6\left(63 c^{8} x^{8}+237 x^{6} c^{6}+334 c^{4} x^{4}+209 c^{2} x^{2}+49\right) d^{3}}+\frac{a x^{5}}{2 c^{2} d\left(c^{2} d x^{2}+d\right)^{3 / 2}}+\frac{5 a x^{3}}{6 c^{4} d\left(c^{2} d x^{2}+d\right)^{3 / 2}}+\frac{5 a x}{2 c^{6} d^{2} \sqrt{c^{2} d x^{2}+d}} \\
& -\frac{5 a \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{2 c^{6} d^{2} \sqrt{c^{2} d}}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)}}{8 c^{7} d^{3} \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Problem 48: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}(a+b \operatorname{arcsinh}(c x))}{\left(c^{2} d x^{2}+d\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 179 leaves, 7 steps):

$$
-\frac{x^{3}(a+b \operatorname{arcsinh}(c x))}{3 c^{2} d\left(c^{2} d x^{2}+d\right)^{3 / 2}}-\frac{x(a+b \operatorname{arcsinh}(c x))}{c^{4} d^{2} \sqrt{c^{2} d x^{2}+d}}+\frac{b}{6 c^{5} d^{2} \sqrt{c^{2} x^{2}+1} \sqrt{c^{2} d x^{2}+d}}+\frac{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}}{2 b c^{5} d^{2} \sqrt{c^{2} d x^{2}+d}}+\frac{2 b \ln \left(c^{2} x^{2}+1\right) \sqrt{c^{2} x^{2}+1}}{3 c^{5} d^{2} \sqrt{c^{2} d x^{2}+d}}
$$

Result(type 3, 1429 leaves):

$$
\begin{aligned}
& -\frac{a x^{3}}{3 c^{2} d\left(c^{2} d x^{2}+d\right)^{3 / 2}}-\frac{a x}{c^{4} d^{2} \sqrt{c^{2} d x^{2}+d}}+\frac{a \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{c^{4} d^{2} \sqrt{c^{2} d}}+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2}}{2 \sqrt{c^{2} x^{2}+1} c^{5} d^{3}}-\frac{8 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh(cx)}}{3 \sqrt{c^{2} x^{2}+1} c^{5} d^{3}} \\
& -\frac{32 b \sqrt{d\left(c^{2} x^{2}+1\right)} c^{2} \operatorname{arcsinh}(c x) x^{7}}{\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3}}+\frac{32 b \sqrt{d\left(c^{2} x^{2}+1\right)} c \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1} x^{6}}{\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3}} \\
& -\frac{8 b \sqrt{d\left(c^{2} x^{2}+1\right)} c^{2} x^{7}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3}}+\frac{8 b \sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+1\right) x^{5}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3}} \\
& -\frac{76 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{5}}{\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3}}+\frac{84 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh(cx)\sqrt {c^{2}x^{2}+1}x^{4}}}{\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c} \\
& -\frac{22 b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{5}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3}}+\frac{4 b \sqrt{d\left(c^{2} x^{2}+1\right)} \sqrt{c^{2} x^{2}+1} x^{4}}{\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c} \\
& +\frac{14 b \sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+1\right) x^{3}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{2}}-\frac{181 b \sqrt{d\left(c^{2} x^{2}+1\right) \operatorname{arcsinh}(c x) x^{3}}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{2}} \\
& +\frac{220 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1} x^{2}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{3}}-\frac{20 b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{3}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{13 b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{2} \sqrt{c^{2} x^{2}+1}}{2\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{3}}+\frac{2 b \sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+1\right) x}{\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{4}} \\
& -\frac{16 b \sqrt{d\left(c^{2} x^{2}+1\right)} x \operatorname{arcsinh}(c x)}{\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{4}}+\frac{64 b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{5}} \\
& -\frac{2 b \sqrt{d\left(c^{2} x^{2}+1\right)} x}{\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{4}}+\frac{8 b \sqrt{d\left(c^{2} x^{2}+1\right)} \sqrt{c^{2} x^{2}+1}}{3\left(24 c^{8} x^{8}+87 x^{6} c^{6}+118 c^{4} x^{4}+71 c^{2} x^{2}+16\right) d^{3} c^{5}} \\
& +\frac{4 b \sqrt{d\left(c^{2} x^{2}+1\right)} \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 \sqrt{c^{2} x^{2}+1} c^{5} d^{3}}
\end{aligned}
$$

Problem 51: Unable to integrate problem.

$$
\int \frac{x^{m} \operatorname{arcsinh}(a x)}{\sqrt{a^{2} x^{2}+1}} \mathrm{~d} x
$$

Optimal(type 5, 88 leaves, 1 step):

$$
\frac{x^{1+m} \operatorname{arcsinh}(a x) \text { hypergeom }\left(\left[\frac{1}{2}, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-a^{2} x^{2}\right)}{1+m}-\frac{a x^{2+m} \text { HypergeometricPFQ }\left(\left[1,1+\frac{m}{2}, 1+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}\right],-a^{2} x^{2}\right)}{m^{2}+3 m+2}
$$

Result(type 8, 21 leaves):

$$
\int \frac{x^{m} \operatorname{arcsinh}(a x)}{\sqrt{a^{2} x^{2}+1}} \mathrm{~d} x
$$

Problem 53: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c^{2} d x^{2}+d\right)(a+b \operatorname{arcsinh}(c x))^{2}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 4, 197 leaves, 10 steps):
$\frac{c^{2} d(a+b \operatorname{arcsinh}(c x))^{2}}{2}-\frac{d\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{2 x^{2}}+\frac{c^{2} d(a+b \operatorname{arcsinh}(c x))^{3}}{3 b}+c^{2} d(a+b \operatorname{arcsinh}(c x))^{2} \ln \left(1-\frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right)$
$+b^{2} c^{2} d \ln (x)-b c^{2} d(a+b \operatorname{arcsinh}(c x)) \operatorname{poly} \log \left(2, \frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right)-\frac{b^{2} c^{2} d \operatorname{poly} \log \left(3, \frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right)}{2}$
$-\frac{b c d(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{x}$
Result(type 4, 514 leaves):
$c^{2} d a^{2} \ln (c x)-\frac{d a^{2}}{2 x^{2}}-\frac{c^{2} d b^{2} \operatorname{arcsinh}(c x)^{3}}{3}-\frac{c d b^{2} \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1}}{x}+c^{2} d b^{2} \operatorname{arcsinh}(c x)-\frac{d b^{2} \operatorname{arcsinh}(c x)^{2}}{2 x^{2}}+c^{2} d b^{2} \ln \left(c x+\sqrt{c^{2} x^{2}+1}-1\right)$

$$
-2 c^{2} d b^{2} \ln \left(c x+\sqrt{c^{2} x^{2}+1}\right)+c^{2} d b^{2} \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)+c^{2} d b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)+2 c^{2} d b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}(2,-c x
$$

$$
\left.-\sqrt{c^{2} x^{2}+1}\right)-2 c^{2} d b^{2} \text { polylog }\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right)+c^{2} d b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)+2 c^{2} d b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}(2, c x
$$

$$
\left.+\sqrt{c^{2} x^{2}+1}\right)-2 c^{2} d b^{2} \operatorname{polylog}\left(3, c x+\sqrt{c^{2} x^{2}+1}\right)-c^{2} d a b \operatorname{arcsinh}(c x)^{2}-\frac{c d a b \sqrt{c^{2} x^{2}+1}}{x}+c^{2} d a b-\frac{d a b \operatorname{arcsinh}(c x)}{x^{2}}
$$

$$
+2 c^{2} d a b \operatorname{arcsinh}(c x) \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)+2 c^{2} d a b \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)+2 c^{2} d a b \operatorname{arcsinh}(c x) \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)
$$

$$
+2 c^{2} d a b \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)
$$

Problem 58: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c^{2} d x^{2}+d\right)^{3}(a+b \operatorname{arcsinh}(c x))^{2}}{x} \mathrm{~d} x
$$

Optimal(type 4, 334 leaves, 26 steps):

$$
\begin{aligned}
& \frac{71 b^{2} c^{2} d^{3} x^{2}}{144}+\frac{7 b^{2} c^{4} d^{3} x^{4}}{144}+\frac{b^{2} d^{3}\left(c^{2} x^{2}+1\right)^{3}}{108}-\frac{7 b c d^{3} x\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{36}-\frac{b c d^{3} x\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))}{18} \\
& -\frac{19 d^{3}(a+b \operatorname{arcsinh}(c x))^{2}}{48}+\frac{d^{3}\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{2}+\frac{d^{3}\left(c^{2} x^{2}+1\right)^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{4}+\frac{d^{3}\left(c^{2} x^{2}+1\right)^{3}(a+b \operatorname{arcsinh}(c x))^{2}}{6} \\
& +\frac{d^{3}(a+b \operatorname{arcsinh}(c x))^{3}}{3 b}+d^{3}(a+b \operatorname{arcsinh}(c x))^{2} \ln \left(1-\frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right)-b d^{3}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right) \\
& \quad b^{2} d^{3} \operatorname{polylog}\left(3, \frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right) \\
& -\frac{19 b c d^{3} x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{24}
\end{aligned}
$$

Result(type 4, 705 leaves):
$2 d^{3} a b$ polylog$\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)+2 d^{3} a b \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)+\frac{25 d^{3} a b \operatorname{arcsinh}(c x)}{24}-d^{3} a b \operatorname{arcsinh}(c x)^{2}+2 d^{3} b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}(2, c x$
$\left.+\sqrt{c^{2} x^{2}+1}\right)+2 d^{3} b^{2} \operatorname{arcsinh}(c x)$ polylog $\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)+d^{3} b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)+d^{3} b^{2} \operatorname{arcsinh}(c x)^{2} \ln (1+c x$
$\left.+\sqrt{c^{2} x^{2}+1}\right)+\frac{d^{3} a^{2} c^{6} x^{6}}{6}+\frac{3 d^{3} a^{2} c^{4} x^{4}}{4}+\frac{3 d^{3} a^{2} c^{2} x^{2}}{2}+\frac{d^{3} b^{2} c^{6} x^{6}}{108}+d^{3} a^{2} \ln (c x)-\frac{d^{3} b^{2} \operatorname{arcsinh}(c x)^{3}}{3}-2 d^{3} b^{2} \operatorname{polylog}\left(3, c x+\sqrt{c^{2} x^{2}+1}\right)$
$-2 d^{3} b^{2}$ polylog $\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right)+\frac{25 d^{3} b^{2} \operatorname{arcsinh}(c x)^{2}}{48}+2 d^{3} a b \operatorname{arcsinh}(c x) \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)+2 d^{3} a b \operatorname{arcsinh}(c x) \ln (1+c x$
$\left.+\sqrt{c^{2} x^{2}+1}\right)+\frac{25 b^{2} c^{2} d^{3} x^{2}}{48}+\frac{11 b^{2} c^{4} d^{3} x^{4}}{144}+\frac{d^{3} b^{2} \operatorname{arcsinh}(c x)^{2} c^{6} x^{6}}{6}+\frac{3 d^{3} b^{2} \operatorname{arcsinh}(c x)^{2} c^{4} x^{4}}{4}+\frac{3 d^{3} b^{2} \operatorname{arcsinh}(c x)^{2} c^{2} x^{2}}{2}$
$+\frac{d^{3} a b \operatorname{arcsinh}(c x) c^{6} x^{6}}{3}+3 d^{3} a b \operatorname{arcsinh}(c x) c^{2} x^{2}+\frac{3 d^{3} a b \operatorname{arcsinh}(c x) c^{4} x^{4}}{2}-\frac{25 d^{3} a b c x \sqrt{c^{2} x^{2}+1}}{24}-\frac{d^{3} a b c^{5} x^{5} \sqrt{c^{2} x^{2}+1}}{18}$

$$
-\frac{11 d^{3} a b c^{3} x^{3} \sqrt{c^{2} x^{2}+1}}{36}-\frac{25 d^{3} b^{2} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) c x}{24}-\frac{d^{3} b^{2} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) c^{5} x^{5}}{18}-\frac{11 d^{3} b^{2} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x) c^{3} x^{3}}{36}
$$

$$
+\frac{811 d^{3} b^{2}}{3456}
$$

Problem 59: Unable to integrate problem.

$$
\int \frac{x^{4}(a+b \operatorname{arcsinh}(c x))^{2}}{c^{2} d x^{2}+d} \mathrm{~d} x
$$

Optimal(type 4, 300 leaves, 16 steps):

$$
\begin{aligned}
& -\frac{22 b^{2} x}{9 c^{4} d}+\frac{2 b^{2} x^{3}}{27 c^{2} d}-\frac{x(a+b \operatorname{arcsinh}(c x))^{2}}{c^{4} d}+\frac{x^{3}(a+b \operatorname{arcsinh}(c x))^{2}}{3 c^{2} d}+\frac{2(a+b \operatorname{arcsinh}(c x))^{2} \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c^{5} d} \\
& \quad-\frac{2 \mathrm{I} b(a+b \operatorname{arcsinh}(c x)) \operatorname{poly} \log \left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{c^{5} d}+\frac{2 \mathrm{I} b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{c^{5} d} \\
& \quad+\frac{2 \mathrm{I} b^{2} \operatorname{polylog}\left(3,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{c^{5} d}-\frac{2 \mathrm{I} b^{2} \operatorname{polylog}\left(3, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{c^{5} d}+\frac{22 b(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{9 c^{5} d} \\
& \quad-\frac{2 b x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{9 c^{3} d}
\end{aligned}
$$

Result(type 8, 28 leaves):

$$
\int \frac{x^{4}(a+b \operatorname{arcsinh}(c x))^{2}}{c^{2} d x^{2}+d} \mathrm{~d} x
$$

Problem 61: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x\left(c^{2} d x^{2}+d\right)} d x
$$

Optimal(type 4, 157 leaves, 9 steps):
$-\frac{2(a+b \operatorname{arcsinh}(c x))^{2} \operatorname{arctanh}\left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d}-\frac{b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d}$

$$
+\frac{b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d}+\frac{b^{2} \operatorname{polylog}\left(3,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d}-\frac{b^{2} \operatorname{polylog}\left(3,\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d}
$$

Result(type 4, 353 leaves):
$\frac{a^{2} \ln (c x)}{d}-\frac{a^{2} \ln \left(c^{2} x^{2}+1\right)}{2 d}+\frac{b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)}{d}+\frac{2 b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)}{d}$

$$
\begin{aligned}
& -\frac{2 b^{2} \text { polylog }\left(3, c x+\sqrt{c^{2} x^{2}+1}\right)}{d}-\frac{b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d}-\frac{b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d} \\
& +\frac{b^{2} \operatorname{poly} \log \left(3,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d}+\frac{b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)}{d}+\frac{2 b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d} \\
& -\frac{2 b^{2} \text { polylog }\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d}+\frac{2 a b \operatorname{dilog}\left(\frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right)}{d}-\frac{a b \operatorname{dilog}\left(\frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{4}}\right)}{2 d}
\end{aligned}
$$

Problem 62: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{2}\left(c^{2} d x^{2}+d\right)} d x
$$

Optimal(type 4, 269 leaves, 15 steps):
$-\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{d x}-\frac{2 c(a+b \operatorname{arcsinh}(c x))^{2} \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{d}-\frac{4 b c(a+b \operatorname{arcsinh}(c x)) \operatorname{arctanh}\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{d}$
$-\frac{2 b^{2} c \text { polylog }\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d}+\frac{2 \mathrm{I} b c(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d}$
$-\frac{2 \mathrm{I} b c(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d}+\frac{2 b^{2} c \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)}{d}-\frac{2 \mathrm{I} b^{2} c \operatorname{polylog}\left(3,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d}$
$+\frac{2 \mathrm{I} b^{2} c \operatorname{polylog}\left(3, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d}$
Result(type 8, 28 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{2}\left(c^{2} d x^{2}+d\right)} \mathrm{d} x
$$

Problem 63: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{4}\left(c^{2} d x^{2}+d\right)} d x
$$

Optimal(type 4, 348 leaves, 24 steps):
$-\frac{b^{2} c^{2}}{3 d x}-\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{3 d x^{3}}+\frac{c^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{d x}+\frac{2 c^{3}(a+b \operatorname{arcsinh}(c x))^{2} \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{d}$

$$
\begin{aligned}
& +\frac{14 b c^{3}(a+b \operatorname{arcsinh}(c x)) \operatorname{arctanh}\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{3 d}+\frac{7 b^{2} c^{3} \operatorname{poly} \log \left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)}{3 d} \\
& -\frac{2 \mathrm{I} b c^{3}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d}+\frac{2 \mathrm{I} b c^{3}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d} \\
& -\frac{7 b^{2} c^{3} \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)}{3 d}+\frac{2 \mathrm{I} b^{2} c^{3} \operatorname{polylog}\left(3,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d}-\frac{2 \mathrm{I} b^{2} c^{3} \operatorname{polylog}\left(3, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d} \\
& -\frac{b c(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{3 d x^{2}} \\
& \text { Result (type 8, 28 leaves): } \\
& \quad \int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{4}\left(c^{2} d x^{2}+d\right)} \mathrm{d} x
\end{aligned}
$$

Problem 64: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{\left(c^{2} d x^{2}+d\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 243 leaves, 11 steps):
$\frac{x(a+b \operatorname{arcsinh}(c x))^{2}}{2 d^{2}\left(c^{2} x^{2}+1\right)}+\frac{(a+b \operatorname{arcsinh}(c x))^{2} \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d^{2}}-\frac{b^{2} \arctan (c x)}{c d^{2}}-\frac{\mathrm{I} b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{c d^{2}}$

$$
+\frac{\mathrm{I} b(a+b \operatorname{arcsinh}(c x)) \operatorname{poly} \log \left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{c d^{2}}+\frac{\mathrm{I} b^{2} \operatorname{polylog}\left(3,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{c d^{2}}-\frac{\mathrm{I} b^{2} \operatorname{poly} \log \left(3, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{c d^{2}}
$$

$$
+\frac{b(a+b \operatorname{arcsinh}(c x))}{c d^{2} \sqrt{c^{2} x^{2}+1}}
$$

Result(type 8, 25 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{\left(c^{2} d x^{2}+d\right)^{2}} \mathrm{~d} x
$$

Problem 65: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x\left(c^{2} d x^{2}+d\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 228 leaves, 12 steps):
$\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{2 d^{2}\left(c^{2} x^{2}+1\right)}-\frac{2(a+b \operatorname{arcsinh}(c x))^{2} \operatorname{arctanh}\left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{2}}+\frac{b^{2} \ln \left(c^{2} x^{2}+1\right)}{2 d^{2}}$

$$
\begin{aligned}
& -\frac{b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{2}}+\frac{b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{2}} \\
& +\frac{b^{2} \operatorname{polylog}\left(3,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d^{2}}-\frac{b^{2} \operatorname{polylog}\left(3,\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d^{2}}-\frac{b c x(a+b \operatorname{arcsinh}(c x))}{d^{2} \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 4, 723 leaves):

$$
\frac{a b \operatorname{arcsinh}(c x)}{d^{2}\left(c^{2} x^{2}+1\right)}-\frac{2 a b \operatorname{arcsinh}(c x) \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{2}}+\frac{2 a b \operatorname{arcsinh}(c x) \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}
$$

$$
+\frac{2 a b \operatorname{arcsinh}(c x) \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}+\frac{a^{2}}{2 d^{2}\left(c^{2} x^{2}+1\right)}+\frac{a^{2} \ln (c x)}{d^{2}}-\frac{a^{2} \ln \left(c^{2} x^{2}+1\right)}{2 d^{2}}-\frac{2 b^{2} \ln \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}
$$

$$
+\frac{b^{2} \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{2}}-\frac{2 b^{2} \text { polylog }\left(3, c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}-\frac{2 b^{2} \text { polylog }\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}+\frac{a b}{d^{2}\left(c^{2} x^{2}+1\right)}
$$

$$
-\frac{a b \text { polylog }\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{2}}+\frac{2 a b \text { polylog }\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}+\frac{2 a b \text { polylog }\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}+\frac{b^{2} \operatorname{arcsinh}(c x)^{2}}{2 d^{2}\left(c^{2} x^{2}+1\right)}
$$

$$
+\frac{b^{2} \operatorname{arcsinh}(c x)}{d^{2}\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}+\frac{2 b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}
$$

$$
-\frac{b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{2}}-\frac{b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{2}}+\frac{b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}
$$

$$
+\frac{2 b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{2}}+\frac{b^{2} \operatorname{polylog}\left(3,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d^{2}}+\frac{a b c^{2} x^{2}}{d^{2}\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \operatorname{arcsinh}(c x) c^{2} x^{2}}{d^{2}\left(c^{2} x^{2}+1\right)}
$$

$$
-\frac{b^{2} \operatorname{arcsinh}(c x) c x}{d^{2} \sqrt{c^{2} x^{2}+1}}-\frac{a b c x}{d^{2} \sqrt{c^{2} x^{2}+1}}
$$

Problem 66: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{4}\left(c^{2} d x^{2}+d\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 444 leaves, 32 steps):

$$
\begin{aligned}
& -\frac{b^{2} c^{2}}{3 d^{2} x}-\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{3 d^{2} x^{3}\left(c^{2} x^{2}+1\right)}+\frac{5 c^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 d^{2} x\left(c^{2} x^{2}+1\right)}+\frac{5 c^{4} x(a+b \operatorname{arcsinh}(c x))^{2}}{2 d^{2}\left(c^{2} x^{2}+1\right)}+\frac{5 c^{3}(a+b \operatorname{arcsinh}(c x))^{2} \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{2}} \\
& \quad-\frac{b^{2} c^{3} \arctan (c x)}{d^{2}}+\frac{26 b c^{3}(a+b \operatorname{arcsinh}(c x)) \operatorname{arctanh}\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{3 d^{2}}+\frac{13 b^{2} c^{3} \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)}{3 d^{2}}
\end{aligned}
$$

```
\(-\frac{5 \mathrm{I} b c^{3}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d^{2}}+\frac{5 \mathrm{I} b c^{3}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d^{2}}\)
\(-\frac{13 b^{2} c^{3} \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)}{3 d^{2}}+\frac{51 b^{2} c^{3} \text { polylog }\left(3,-1\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d^{2}}-\frac{51 b^{2} c^{3} \text { polylog }\left(3, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{d^{2}}\)
\(+\frac{2 b c^{3}(a+b \operatorname{arcsinh}(c x))}{3 d^{2} \sqrt{c^{2} x^{2}+1}}-\frac{b c(a+b \operatorname{arcsinh}(c x))}{3 d^{2} x^{2} \sqrt{c^{2} x^{2}+1}}\)
```

Result(type 8, 28 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{4}\left(c^{2} d x^{2}+d\right)^{2}} \mathrm{~d} x
$$

Problem 67: Unable to integrate problem.

$$
\int \frac{x^{4}(a+b \operatorname{arcsinh}(c x))^{2}}{\left(c^{2} d x^{2}+d\right)^{3}} \mathrm{~d} x
$$

Optimal(type 4, 331 leaves, 16 steps):

$$
\begin{aligned}
& -\frac{b^{2} x}{12 c^{4} d^{3}\left(c^{2} x^{2}+1\right)}+\frac{b(a+b \operatorname{arcsinh}(c x))}{6 c^{5} d^{3}\left(c^{2} x^{2}+1\right)^{3 / 2}}-\frac{x^{3}(a+b \operatorname{arcsinh}(c x))^{2}}{4 c^{2} d^{3}\left(c^{2} x^{2}+1\right)^{2}}-\frac{3 x(a+b \operatorname{arcsinh}(c x))^{2}}{8 c^{4} d^{3}\left(c^{2} x^{2}+1\right)}+\frac{3(a+b \operatorname{arcsinh}(c x))^{2} \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{4 c^{5} d^{3}} \\
& +\frac{7 b^{2} \arctan (c x)}{6 c^{5} d^{3}}-\frac{3 \mathrm{I} b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{4 c^{5} d^{3}}+\frac{3 \mathrm{I} b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{4 c^{5} d^{3}} \\
& +\frac{3 \mathrm{I} b^{2} \operatorname{polylog}\left(3,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{4 c^{5} d^{3}}-\frac{3 \mathrm{I} b^{2} \operatorname{polylog}\left(3, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{4 c^{5} d^{3}}-\frac{5 b(a+b \operatorname{arcsinh}(c x))}{4 c^{5} d^{3} \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 8, 28 leaves):

$$
\int \frac{x^{4}(a+b \operatorname{arcsinh}(c x))^{2}}{\left(c^{2} d x^{2}+d\right)^{3}} \mathrm{~d} x
$$

Problem 68: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{\left(c^{2} d x^{2}+d\right)^{3}} \mathrm{~d} x
$$

Optimal(type 4, 320 leaves, 15 steps):
$-\frac{b^{2} x}{12 d^{3}\left(c^{2} x^{2}+1\right)}+\frac{b(a+b \operatorname{arcsinh}(c x))}{6 c d^{3}\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{x(a+b \operatorname{arcsinh}(c x))^{2}}{4 d^{3}\left(c^{2} x^{2}+1\right)^{2}}+\frac{3 x(a+b \operatorname{arcsinh}(c x))^{2}}{8 d^{3}\left(c^{2} x^{2}+1\right)}+\frac{3(a+b \operatorname{arcsinh}(c x))^{2} \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{4 c d^{3}}$

$$
\begin{aligned}
& -\frac{5 b^{2} \arctan (c x)}{6 c d^{3}}-\frac{3 \mathrm{I} b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{4 c d^{3}}+\frac{3 \mathrm{I} b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{4 c d^{3}} \\
& +\frac{3 \mathrm{I} b^{2} \operatorname{polylog}\left(3,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{4 c d^{3}}-\frac{3 \mathrm{I} b^{2} \operatorname{polylog}\left(3, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{4 c d^{3}}+\frac{3 b(a+b \operatorname{arcsinh}(c x))}{4 c d^{3} \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 8, 25 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{\left(c^{2} d x^{2}+d\right)^{3}} \mathrm{~d} x
$$

Problem 69: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{3}\left(c^{2} d x^{2}+d\right)^{3}} \mathrm{~d} x
$$

Optimal(type 4, 402 leaves, 23 steps):

$$
\begin{aligned}
& \frac{b^{2} c^{2}}{12 d^{3}\left(c^{2} x^{2}+1\right)}-\frac{b c(a+b \operatorname{arcsinh}(c x))}{d^{3} x\left(c^{2} x^{2}+1\right)^{3 / 2}}-\frac{5 b c^{3} x(a+b \operatorname{arcsinh}(c x))}{6 d^{3}\left(c^{2} x^{2}+1\right)^{3 / 2}}-\frac{3 c^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{4 d^{3}\left(c^{2} x^{2}+1\right)^{2}}-\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{2 d^{3} x^{2}\left(c^{2} x^{2}+1\right)^{2}} \\
& -\frac{3 c^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{2 d^{3}\left(c^{2} x^{2}+1\right)}+\frac{6 c^{2}(a+b \operatorname{arcsinh}(c x))^{2} \operatorname{arctanh}\left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{3}}+\frac{b^{2} c^{2} \ln (x)}{d^{3}}-\frac{7 b^{2} c^{2} \ln \left(c^{2} x^{2}+1\right)}{6 d^{3}} \\
& +\frac{3 b c^{2}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{3}}-\frac{\left.3 b c^{2}(a+b \operatorname{arcsinh}(c x))\right) \operatorname{polylog}\left(2,\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{3}} \\
& -\frac{3 b^{2} c^{2} \operatorname{polylog}\left(3,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d^{3}}+\frac{3 b^{2} c^{2} \operatorname{polylog}\left(3,\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d^{3} x(a+b \operatorname{arcsinh}(c x))} \\
& 3 d^{3} \sqrt{c^{2} x^{2}+1}
\end{aligned}
$$

Result(type 4, 1435 leaves):

$$
\begin{aligned}
& -\frac{9 c^{2} b^{2} \operatorname{arcsinh}(c x)^{2}}{4 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}-\frac{4 c^{2} b^{2} \operatorname{arcsinh}(c x)}{3 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}-\frac{3 c^{2} b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{3}}-\frac{6 c^{2} b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{3}} \\
& +\frac{3 c^{2} b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{3}}+\frac{3 c^{2} b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{3}} \\
& \quad-\frac{3 c^{2} b^{2} \operatorname{arcsinh}(c x)^{2} \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{3}}-\frac{6 c^{2} b^{2} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{3}}-\frac{b^{2} \operatorname{arcsinh}(c x)^{2}}{2 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right) x^{2}} \\
& \quad+\frac{c^{4} b^{2} x^{2}}{12 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}-\frac{4 c^{2} a b}{3 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}+\frac{3 c^{2} a b \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{d^{3}}-\frac{6 c^{2} a b \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{3}} \\
& -\frac{6 c^{2} a b \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{3}}-\frac{c^{2} a^{2}}{4 d^{3}\left(c^{2} x^{2}+1\right)^{2}}-\frac{c^{2} a^{2}}{d^{3}\left(c^{2} x^{2}+1\right)}+\frac{c^{2} b^{2}}{12 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}+\frac{6 c^{2} b^{2} \operatorname{polylog}\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{3}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3 c^{2} a^{2} \ln (c x)}{d^{3}}+\frac{3 c^{2} a^{2} \ln \left(c^{2} x^{2}+1\right)}{2 d^{3}}+\frac{c^{2} b^{2} \ln \left(c x+\sqrt{c^{2} x^{2}+1}-1\right)}{d^{3}}+\frac{8 c^{2} b^{2} \ln \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{3 d^{3}}-\frac{7 c^{2} b^{2} \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 d^{3}} \\
& +\frac{c^{2} b^{2} \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{3}}+\frac{6 c^{2} b^{2} \operatorname{polylog}\left(3, c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{3}}-\frac{a^{2}}{2 d^{3} x^{2}}-\frac{3 b^{2} c^{2} \operatorname{polylog}\left(3,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{2 d^{3}} \\
& +\frac{4 c^{5} a b x^{3} \sqrt{c^{2} x^{2}+1}}{3 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}-\frac{3 c^{4} a b x^{2} \operatorname{arcsinh}(c x)}{d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}+\frac{c^{3} a b x \sqrt{c^{2} x^{2}+1}}{2 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}-\frac{c a b \sqrt{c^{2} x^{2}+1}}{d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right) x}+\frac{4 c^{5} b^{2} x^{3} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)}{3 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)} \\
& +\frac{d^{3} b^{2} x \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)}{2 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}-\frac{c b^{2} \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)}{d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right) x}-\frac{4 c^{4} a b x^{4}}{3 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}-\frac{\left.8 c^{4}\right)}{3 d^{3}\left(c^{4} x^{4}+2 x^{2} x^{2}+1\right)}-\frac{a b \operatorname{arcsinh}(c x)}{d^{3}\left(c^{4} x^{4}+2 c^{2}+1\right) x^{2}} \\
& -\frac{6 c^{2} a b \operatorname{arcsinh}(c x) \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right)}{d^{3}}-\frac{9 c^{2} a b \operatorname{arcsinh}(c x)}{2 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)} \\
& +\frac{6 c^{2} a b \operatorname{arcsinh}(c x) \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right)}{d^{3}\left(c^{4} x^{4}+2 c^{3} c^{2} x^{2}+1\right)}-\frac{3 c^{4} b^{2} x^{2} \operatorname{arcsinh}(c x)^{2}}{2 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}-\frac{8 c^{4} b^{2} x^{2} \operatorname{arcsinh}(c x)}{3 d^{3}\left(c^{4} x^{4}+2 c^{2} x^{2}+1\right)}
\end{aligned}
$$

Problem 70: Result more than twice size of optimal antiderivative.

$$
\int x^{3}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d} \mathrm{~d} x
$$

Optimal(type 3, 310 leaves, 14 steps):

$$
\begin{aligned}
& -\frac{52 b^{2} \sqrt{c^{2} d x^{2}+d}}{225 c^{4}}-\frac{26 b^{2}\left(c^{2} x^{2}+1\right) \sqrt{c^{2} d x^{2}+d}}{675 c^{4}}+\frac{2 b^{2}\left(c^{2} x^{2}+1\right)^{2} \sqrt{c^{2} d x^{2}+d}}{125 c^{4}}-\frac{2(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{15 c^{4}} \\
& +\frac{x^{2}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{15 c^{2}}+\frac{x^{4}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{5}+\frac{4 a b x \sqrt{c^{2} d x^{2}+d}}{15 c^{3} \sqrt{c^{2} x^{2}+1}}+\frac{4 b^{2} x \operatorname{arcsinh}(c x) \sqrt{c^{2} d x^{2}+d}}{15 c^{3} \sqrt{c^{2} x^{2}+1}} \\
& -\frac{2 b x^{3}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{45 c \sqrt{c^{2} x^{2}+1}}-\frac{2 b c x^{5}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{25 \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 3, 1161 leaves):
$a^{2}\left(\frac{x^{2}\left(c^{2} d x^{2}+d\right)^{3 / 2}}{5 c^{2} d}-\frac{2\left(c^{2} d x^{2}+d\right)^{3 / 2}}{15 d c^{4}}\right)$
$+b^{2}\left(\frac{1}{4000\left(c^{2} x^{2}+1\right) c^{4}}\left(\sqrt{d\left(c^{2} x^{2}+1\right)}\left(16 x^{6} c^{6}+16 \sqrt{c^{2} x^{2}+1} x^{5} c^{5}+28 c^{4} x^{4}+20 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+13 c^{2} x^{2}+5 \sqrt{c^{2} x^{2}+1} c x\right.\right.\right.$
$\left.+1)\left(25 \operatorname{arcsinh}(c x)^{2}-10 \operatorname{arcsinh}(c x)+2\right)\right)$

$$
\begin{aligned}
& -\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}+4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}+3 \sqrt{c^{2} x^{2}+1} c x+1\right)\left(9 \operatorname{arcsinh}(c x)^{2}-6 \operatorname{arcsinh}(c x)+2\right)}{864\left(c^{2} x^{2}+1\right) c^{4}} \\
& -\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+\sqrt{c^{2} x^{2}+1} c x+1\right)\left(\operatorname{arcsinh}(c x)^{2}-2 \operatorname{arcsinh}(c x)+2\right)}{16\left(c^{2} x^{2}+1\right) c^{4}} \\
& -\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}-\sqrt{c^{2} x^{2}+1} c x+1\right)\left(\operatorname{arcsinh}(c x)^{2}+2 \operatorname{arcsinh}(c x)+2\right)}{16\left(c^{2} x^{2}+1\right) c^{4}} \\
& -\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}-4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}-3 \sqrt{c^{2} x^{2}+1} c x+1\right)\left(9 \operatorname{arcsinh}(c x)^{2}+6 \operatorname{arcsinh}(c x)+2\right)}{864\left(c^{2} x^{2}+1\right) c^{4}} \\
& +\frac{1}{4000\left(c^{2} x^{2}+1\right) c^{4}}\left(\sqrt { d ( c ^ { 2 } x ^ { 2 } + 1 ) } \left(16 x^{6} c^{6}-16 \sqrt{c^{2} x^{2}+1} x^{5} c^{5}+28 c^{4} x^{4}-20 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+13 c^{2} x^{2}-5 \sqrt{c^{2} x^{2}+1} c x\right.\right. \\
& \text { +1) } \left.\left.\left(25 \operatorname{arcsinh}(c x)^{2}+10 \operatorname{arcsinh}(c x)+2\right)\right)\right) \\
& +2 a b\left(\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(16 x^{6} c^{6}+16 \sqrt{c^{2} x^{2}+1} x^{5} c^{5}+28 c^{4} x^{4}+20 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+13 c^{2} x^{2}+5 \sqrt{c^{2} x^{2}+1} c x+1\right)(-1+5 \operatorname{arcsinh}(c x))}{800 c^{4}\left(c^{2} x^{2}+1\right)}\right. \\
& -\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}+4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}+3 \sqrt{c^{2} x^{2}+1} c x+1\right)(-1+3 \operatorname{arcsinh}(c x))}{288 c^{4}\left(c^{2} x^{2}+1\right)} \\
& -\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}+\sqrt{c^{2} x^{2}+1} c x+1\right)(-1+\operatorname{arcsinh}(c x))}{16 c^{4}\left(c^{2} x^{2}+1\right)}-\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(c^{2} x^{2}-\sqrt{c^{2} x^{2}+1} c x+1\right)(1+\operatorname{arcsinh}(c x))}{16 c^{4}\left(c^{2} x^{2}+1\right)} \\
& -\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(4 c^{4} x^{4}-4 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+5 c^{2} x^{2}-3 \sqrt{c^{2} x^{2}+1} c x+1\right)(1+3 \operatorname{arcsinh}(c x))}{288 c^{4}\left(c^{2} x^{2}+1\right)} \\
& \left.+\frac{\sqrt{d\left(c^{2} x^{2}+1\right)}\left(16 x^{6} c^{6}-16 \sqrt{c^{2} x^{2}+1} x^{5} c^{5}+28 c^{4} x^{4}-20 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+13 c^{2} x^{2}-5 \sqrt{c^{2} x^{2}+1} c x+1\right)(1+5 \operatorname{arcsinh}(c x))}{800 c^{4}\left(c^{2} x^{2}+1\right)}\right)
\end{aligned}
$$

Problem 71: Result more than twice size of optimal antiderivative.

$$
\int x^{2}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d} \mathrm{~d} x
$$

Optimal(type 3, 251 leaves, 10 steps):
$\frac{b^{2} x \sqrt{c^{2} d x^{2}+d}}{64 c^{2}}+\frac{b^{2} x^{3} \sqrt{c^{2} d x^{2}+d}}{32}+\frac{x(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{8 c^{2}}+\frac{x^{3}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{4}-\frac{b^{2} \operatorname{arcsinh}(c x) \sqrt{c^{2} d x^{2}+d}}{64 c^{3} \sqrt{c^{2} x^{2}+1}}$

$$
-\frac{b x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{-}-\frac{b c x^{4}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{-}-(a+b \operatorname{arcsinh}(c x))^{3} \sqrt{c^{2} d x^{2}+d}
$$

$$
8 c \sqrt{c^{2} x^{2}+1} \quad 8 \sqrt{c^{2} x^{2}+1} \quad 24 b c^{3} \sqrt{c^{2} x^{2}+1}
$$

Result(type 3, 700 leaves):

$$
\begin{aligned}
& \frac{a^{2} x\left(c^{2} d x^{2}+d\right)^{3 / 2}}{4 c^{2} d}-\frac{a^{2} x \sqrt{c^{2} d x^{2}+d}}{8 c^{2}}-\frac{a^{2} d \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{8 c^{2} \sqrt{c^{2} d}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{2}}{8 c \sqrt{c^{2} x^{2}+1}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} c \operatorname{arcsinh}(c x) x^{4}}{8 \sqrt{c^{2} x^{2}+1}} \\
& -\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{3}}{24 \sqrt{c^{2} x^{2}+1} c^{3}}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} c^{2} x^{5}}{32\left(c^{2} x^{2}+1\right)}+\frac{3 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} x^{3}}{64\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} x}{64 c^{2}\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} c^{2} \operatorname{arcsinh}(c x)^{2} x^{5}}{4\left(c^{2} x^{2}+1\right)} \\
& +\frac{3 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} x^{3}}{8\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} x \operatorname{arcsinh}(c x)^{2}}{8 c^{2}\left(c^{2} x^{2}+1\right)}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)}{64 c^{3} \sqrt{c^{2} x^{2}+1}}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2}}{8 \sqrt{c^{2} x^{2}+1} c^{3}} \\
& -\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)}}{84}+\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} c^{2} \operatorname{arcsinh}(c x) x^{5}}{2\left(c^{2} x^{2}+1\right)}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} c x^{4}}{8 \sqrt{c^{2} x^{2}+1}}+\frac{3 a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{3}}{4\left(c^{2} x^{2}+1\right)}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} x^{2}}{8 c \sqrt{c^{2} x^{2}+1}} \\
& +\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x}{4 c^{2}\left(c^{2} x^{2}+1\right)}
\end{aligned}
$$

Problem 72: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 4, 371 leaves, 13 steps):

$$
\begin{aligned}
& -\frac{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{2 x^{2}}-\frac{b c(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{x \sqrt{c^{2} x^{2}+1}}-\frac{c^{2}(a+b \operatorname{arcsinh}(c x))^{2} \operatorname{arctanh}\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}} \\
& -\frac{b^{2} c^{2} \operatorname{arctanh}\left(\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}}-\frac{b c^{2}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}} \\
& +\frac{b c^{2}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}}+\frac{b^{2} c^{2} \operatorname{polylog}\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}} \\
& -\frac{b^{2} c^{2} \operatorname{polylog}\left(3, c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 4, 869 leaves):

$$
\begin{array}{r}
-\frac{a^{2}\left(c^{2} d x^{2}+d\right)^{3 / 2}}{2 d x^{2}}-\frac{a^{2} \ln \left(\frac{2 d+2 \sqrt{d} \sqrt{c^{2} d x^{2}+d}}{x}\right) \sqrt{d} c^{2}}{2}+\frac{a^{2} \sqrt{c^{2} d x^{2}+d} c^{2}}{2}-\frac{b^{2} \operatorname{arcsinh}(c x)^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} c^{2}}{2\left(c^{2} x^{2}+1\right)} \\
-\frac{b^{2} \operatorname{arcsinh}(c x) \sqrt{d\left(c^{2} x^{2}+1\right)} c}{x \sqrt{c^{2} x^{2}+1}}-\frac{b^{2} \operatorname{arcsinh}(c x)^{2} \sqrt{d\left(c^{2} x^{2}+1\right)}}{2 x^{2}\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right) c^{2}}{2 \sqrt{c^{2} x^{2}+1}}
\end{array}
$$

$+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right) c^{2}}{\sqrt{c^{2} x^{2}+1}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)}}{\operatorname{polylog}\left(3, c x+\sqrt{c^{2} x^{2}+1}\right) c^{2}}$
$\sqrt{c^{2} x^{2}+1} \sqrt{c^{2} x^{2}+1}$


$$
2 \sqrt{c^{2} x^{2}+1} \quad \sqrt{c^{2} x^{2}+1}
$$

$+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{polylog}\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right) c^{2}}{\sqrt{c^{2} x^{2}+1}}-\frac{2 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arctanh}\left(c x+\sqrt{c^{2} x^{2}+1}\right) c^{2}}{\sqrt{c^{2} x^{2}+1}}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) c^{2}}{c^{2} x^{2}+1}$
$-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} c}{x \sqrt{c^{2} x^{2}+1}}-\frac{a b \operatorname{arcsinh}(c x) \sqrt{d\left(c^{2} x^{2}+1\right)}}{x^{2}\left(c^{2} x^{2}+1\right)}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right) c^{2}}{\sqrt{c^{2} x^{2}+1}}$
$-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)}}{\operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right) c^{2}} \sqrt{\sqrt{c^{2} x^{2}+1}}+\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right) c^{2}}{\sqrt{c^{2} x^{2}+1}}$
$+\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right) c^{2}}{\sqrt{c^{2} x^{2}+1}}$
$\sqrt{c^{2} x^{2}+1}$

Problem 73: Result more than twice size of optimal antiderivative.

$$
\int x^{2}\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2} \mathrm{~d} x
$$

Optimal(type 3, 351 leaves, 17 steps):
$\frac{x^{3}\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{6}-\frac{7 b^{2} d x \sqrt{c^{2} d x^{2}+d}}{1152 c^{2}}+\frac{43 b^{2} d x^{3} \sqrt{c^{2} d x^{2}+d}}{1728}+\frac{b^{2} c^{2} d x^{5} \sqrt{c^{2} d x^{2}+d}}{108}$
$+\frac{d x(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{16 c^{2}}+\frac{d x^{3}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{8}+\frac{7 b^{2} d \operatorname{arcsinh}(c x) \sqrt{c^{2} d x^{2}+d}}{1152 c^{3} \sqrt{c^{2} x^{2}+1}}$
$-\frac{b d x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{16 c \sqrt{c^{2} x^{2}+1}}-\frac{7 b c d x^{4}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{48 \sqrt{c^{2} x^{2}+1}}-\frac{b c^{3} d x^{6}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{18 \sqrt{c^{2} x^{2}+1}}$

$$
-\frac{d(a+b \operatorname{arcsinh}(c x))^{3} \sqrt{c^{2} d x^{2}+d}}{}
$$

$48 b c^{3} \sqrt{c^{2} x^{2}+1}$
Result(type 3, 933 leaves):
$\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{4} \operatorname{arcsinh}(c x) x^{7}}{3\left(c^{2} x^{2}+1\right)}+\frac{65 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d x^{3}}{3456\left(c^{2} x^{2}+1\right)}+\frac{a^{2} x\left(c^{2} d x^{2}+d\right)^{5 / 2}}{6 c^{2} d}-\frac{a^{2} d x \sqrt{c^{2} d x^{2}+d}}{16 c^{2}}-\frac{a^{2} d^{2} \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{16 c^{2} \sqrt{c^{2} d}}$

$$
\begin{aligned}
& +\frac{11 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{2} \operatorname{arcsinh}(c x) x^{5}}{12\left(c^{2} x^{2}+1\right)}+\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} d \operatorname{arcsinh}(c x) x}{8 c^{2}\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{4} x^{7}}{108\left(c^{2} x^{2}+1\right)}+\frac{59 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{2} x^{5}}{1728\left(c^{2} x^{2}+1\right)} \\
& -\frac{7 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d x}{1152 c^{2}\left(c^{2} x^{2}+1\right)}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{3} d}{48 \sqrt{c^{2} x^{2}+1} c^{3}}+\frac{17 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d \operatorname{arcsinh}(c x)^{2} x^{3}}{48\left(c^{2} x^{2}+1\right)}+\frac{7 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d \operatorname{arcsinh}(c x)}{1152 c^{3} \sqrt{c^{2} x^{2}+1}} \\
& +\frac{7 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d}{1152 c^{3} \sqrt{c^{2} x^{2}+1}}-\frac{a^{2} x\left(c^{2} d x^{2}+d\right)^{3 / 2}}{24 c^{2}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{3} \operatorname{arcsinh}(c x) x^{6}}{18 \sqrt{c^{2} x^{2}+1}}-\frac{7 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d c \operatorname{arcsinh}(c x) x^{4}}{48 \sqrt{c^{2} x^{2}+1}} \\
& -\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d \operatorname{arcsinh(cx)x^{2}}}{16 c \sqrt{c^{2} x^{2}+1}}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{3} x^{6}}{18 \sqrt{c^{2} x^{2}+1}}-\frac{7 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d c x^{4}}{48 \sqrt{c^{2} x^{2}+1}}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} d x^{2}}{16 c \sqrt{c^{2} x^{2}+1}} \\
& +\frac{17 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d \operatorname{arcsinh}(c x) x^{3}}{24\left(c^{2} x^{2}+1\right)}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} d}{16 \sqrt{c^{2} x^{2}+1} c^{3}}+\frac{11 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{2} \operatorname{arcsinh}(c x)^{2} x^{5}}{24\left(c^{2} x^{2}+1\right)} \\
& +\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d x \operatorname{arcsinh}(c x)^{2}}{16 c^{2}\left(c^{2} x^{2}+1\right)}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d c^{4} \operatorname{arcsinh(cx)^{2}x^{7}}}{6\left(c^{2} x^{2}+1\right)}
\end{aligned}
$$

Problem 74: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c^{2} d x^{2}+d\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{x} \mathrm{~d} x
$$

Optimal(type 4, 614 leaves, 23 steps):

$$
\begin{aligned}
& \frac{d\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{3}+\frac{\left(c^{2} d x^{2}+d\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{5}+\frac{598 b^{2} d^{2} \sqrt{c^{2} d x^{2}+d}}{225}+\frac{74 b^{2} d^{2}\left(c^{2} x^{2}+1\right) \sqrt{c^{2} d x^{2}+d}}{675} \\
& +\frac{2 b^{2} d^{2}\left(c^{2} x^{2}+1\right)^{2} \sqrt{c^{2} d x^{2}+d}}{125}+d^{2}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}-\frac{2 a b c d^{2} x \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}}-\frac{2 b^{2} c d^{2} x \operatorname{arcsinh}(c x) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}} \\
& \quad-\frac{16 b c d^{2} x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{15 \sqrt{c^{2} x^{2}+1}}-\frac{22 b c^{3} d^{2} x^{3}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{45 \sqrt{c^{2} x^{2}+1}}-\frac{2 b c^{5} d^{2} x^{5}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{25 \sqrt{c^{2} x^{2}+1}} \\
& \quad-\frac{2 d^{2}(a+b \operatorname{arcsinh}(c x))^{2} \operatorname{arctanh}\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}}-\frac{2 b d^{2}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}} \\
& +\frac{2 b d^{2}(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}}+\frac{2 b^{2} d^{2} \operatorname{polylog}\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}} \\
& -\frac{2 b^{2} d^{2} \operatorname{polylog}\left(3, c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} d x^{2}+d}}{\sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 4, 1320 leaves):
$\frac{23 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x)^{2}}{15\left(c^{2} x^{2}+1\right)}+\frac{2 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{polylog}\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}-\frac{2 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{polylog}\left(3, c x+\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}$
$+\frac{2 a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}-\frac{2 a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}$
$+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x)^{2} x^{6} c^{6}}{5\left(c^{2} x^{2}+1\right)}+\frac{14 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x)^{2} x^{4} c^{4}}{15\left(c^{2} x^{2}+1\right)}+\frac{34 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x)^{2} x^{2} c^{2}}{15\left(c^{2} x^{2}+1\right)}$
$-\frac{46 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} c x}{15 \sqrt{c^{2} x^{2}+1}}-\frac{2 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} x^{5} c^{5}}{25 \sqrt{c^{2} x^{2}+1}}-\frac{22 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} x^{3} c^{3}}{45 \sqrt{c^{2} x^{2}+1}}-\frac{2 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x) x^{5} c^{5}}{25 \sqrt{c^{2} x^{2}+1}}$
$-\frac{22 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x) x^{3} c^{3}}{45 \sqrt{c^{2} x^{2}+1}}-\frac{46 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x) x c}{15 \sqrt{c^{2} x^{2}+1}}+\frac{9394 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2}}{3375\left(c^{2} x^{2}+1\right)}+\frac{a^{2} d\left(c^{2} d x^{2}+d\right)^{3 / 2}}{3}$
$-a^{2} d^{5} / 2 \ln \left(\frac{2 d+2 \sqrt{d} \sqrt{c^{2} d x^{2}+d}}{x}\right)+a^{2} \sqrt{c^{2} d x^{2}+d} d^{2}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} \ln \left(1-c x-\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}$
$+\frac{2 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} \ln \left(1+c x+\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}$
$-\frac{2 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}+\frac{2 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} x^{6} c^{6}}{125\left(c^{2} x^{2}+1\right)}+\frac{532 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} x^{4} c^{4}}{3375\left(c^{2} x^{2}+1\right)}$
$+\frac{9872 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} c^{2} x^{2}}{3375\left(c^{2} x^{2}+1\right)}+\frac{2 a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}-\frac{2 a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right) d^{2}}{\sqrt{c^{2} x^{2}+1}}$
$+\frac{46 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x)}{15\left(c^{2} x^{2}+1\right)}+\frac{28 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x) x^{4} c^{4}}{15\left(c^{2} x^{2}+1\right)}+\frac{68 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x) x^{2} c^{2}}{15\left(c^{2} x^{2}+1\right)}$
$+\frac{2 a b \sqrt{d\left(c^{2} x^{2}+1\right)} d^{2} \operatorname{arcsinh}(c x) x^{6} c^{6}}{5\left(c^{2} x^{2}+1\right)}+\frac{\left(c^{2} d x^{2}+d\right)^{5 / 2} a^{2}}{5}$

Problem 75: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c^{2} d x^{2}+d\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{x^{4}} \mathrm{~d} x
$$

Optimal(type 4, 511 leaves, 27 steps):
$-\frac{5 c^{2} d\left(c^{2} d x^{2}+d\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 x}-\frac{\left(c^{2} d x^{2}+d\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 x^{3}}+\frac{7 b^{2} c^{4} d^{2} x \sqrt{c^{2} d x^{2}+d}}{12}-\frac{b^{2} c^{2} d^{2}\left(c^{2} x^{2}+1\right) \sqrt{c^{2} d x^{2}+d}}{3 x}$

```
\(-\frac{b c d^{2}\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{3 x^{2}}+\frac{5 c^{4} d^{2} x(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{2}-\frac{23 b^{2} c^{3} d^{2} \operatorname{arcsinh}(c x) \sqrt{c^{2} d x^{2}+d}}{12 \sqrt{c^{2} x^{2}+1}}\)
\(-\frac{5 b c^{5} d^{2} x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{2 \sqrt{c^{2} x^{2}+1}}+\frac{7 c^{3} d^{2}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{3 \sqrt{c^{2} x^{2}+1}}+\frac{5 c^{3} d^{2}(a+b \operatorname{arcsinh}(c x))^{3} \sqrt{c^{2} d x^{2}+d}}{6 b \sqrt{c^{2} x^{2}+1}}\)
\(+\frac{14 b c^{3} d^{2}(a+b \operatorname{arcsinh}(c x)) \ln \left(1-\frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right) \sqrt{c^{2} d x^{2}+d}}{3 \sqrt{c^{2} x^{2}+1}}-\frac{7 b^{2} c^{3} d^{2} \operatorname{polylog}\left(2, \frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right) \sqrt{c^{2} d x^{2}+d}}{3 \sqrt{c^{2} x^{2}+1}}\)
\(+\frac{7 b c^{3} d^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1} \sqrt{c^{2} d x^{2}+d}}{3}\)
```

Result(type ?, 3310 leaves): Display of huge result suppressed!
Problem 79: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{\sqrt{c^{2} d x^{2}+d}} d x
$$

Optimal(type 3, 41 leaves, 1 step):

$$
\frac{(a+b \operatorname{arcsinh}(c x))^{3} \sqrt{c^{2} x^{2}+1}}{3 b c \sqrt{c^{2} d x^{2}+d}}
$$

Result(type 3, 119 leaves):

$$
\frac{a^{2} \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{\sqrt{c^{2} d}}+\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{3}}{3 \sqrt{c^{2} x^{2}+1} c d}+\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2}}{\sqrt{c^{2} x^{2}+1} c d}
$$

Problem 80: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{4} \sqrt{c^{2} d x^{2}+d}} \mathrm{~d} x
$$

Optimal(type 4, 281 leaves, 9 steps):

$$
\begin{aligned}
& -\frac{b^{2} c^{2}\left(c^{2} x^{2}+1\right)}{3 x \sqrt{c^{2} d x^{2}+d}}-\frac{b c(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{3 x^{2} \sqrt{c^{2} d x^{2}+d}}-\frac{2 c^{3}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}}{3 \sqrt{c^{2} d x^{2}+d}} \\
& \quad-\frac{4 b c^{3}(a+b \operatorname{arcsinh}(c x)) \ln \left(1-\frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right) \sqrt{c^{2} x^{2}+1}}{3 \sqrt{c^{2} d x^{2}+d}}+\frac{2 b^{2} c^{3} \operatorname{polylog}\left(2, \frac{1}{\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}}\right) \sqrt{c^{2} x^{2}+1}}{3 \sqrt{c^{2} d x^{2}+d}}
\end{aligned}
$$

$-\frac{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{3 d x^{3}}+\frac{2 c^{2}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{3 d x}$
Result(type ?, 2146 leaves): Display of huge result suppressed!
Problem 82: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x^{4}\left(c^{2} d x^{2}+d\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 438 leaves, 24 steps):

```
\(-\frac{b^{2} c^{2}\left(c^{2} x^{2}+1\right)}{3 d x \sqrt{c^{2} d x^{2}+d}}-\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{3 d x^{3} \sqrt{c^{2} d x^{2}+d}}+\frac{4 c^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 d x \sqrt{c^{2} d x^{2}+d}}+\frac{8 c^{4} x(a+b \operatorname{arcsinh}(c x))^{2}}{3 d \sqrt{c^{2} d x^{2}+d}}-\frac{b c(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{3 d x^{2} \sqrt{c^{2} d x^{2}+d}}\)
    \(+\frac{8 c^{3}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}}{3 d \sqrt{c^{2} d x^{2}+d}}+\frac{20 b c^{3}(a+b \operatorname{arcsinh}(c x)) \operatorname{arctanh}\left(\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right) \sqrt{c^{2} x^{2}+1}}{3 d \sqrt{c^{2} d x^{2}+d}}\)
    \(-\frac{16 b c^{3}(a+b \operatorname{arcsinh}(c x)) \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right) \sqrt{c^{2} x^{2}+1}}{3 d \sqrt{c^{2} d x^{2}+d}}-\frac{b^{2} c^{3} \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right) \sqrt{c^{2} x^{2}+1}}{d \sqrt{c^{2} d x^{2}+d}}\)
    \(-\frac{5 b^{2} c^{3} \operatorname{polylog}\left(2,\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right) \sqrt{c^{2} x^{2}+1}}{3 d \sqrt{c^{2} d x^{2}+d}}\)
```

Result(type ?, 2608 leaves): Display of huge result suppressed!
Problem 83: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}(a+b \operatorname{arcsinh}(c x))^{2}}{\left(c^{2} d x^{2}+d\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 294 leaves, 16 steps):

$$
\begin{aligned}
& -\frac{x^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 c^{2} d\left(c^{2} d x^{2}+d\right)^{3 / 2}}-\frac{b^{2}}{3 c^{4} d^{2} \sqrt{c^{2} d x^{2}+d}}-\frac{2(a+b \operatorname{arcsinh}(c x))^{2}}{3 c^{4} d^{2} \sqrt{c^{2} d x^{2}+d}}-\frac{b x(a+b \operatorname{arcsinh}(c x))}{3 c^{3} d^{2} \sqrt{c^{2} x^{2}+1} \sqrt{c^{2} d x^{2}+d}} \\
& \quad+\frac{10 b(a+b \operatorname{arcsinh}(c x)) \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} x^{2}+1}}{3 c^{4} d^{2} \sqrt{c^{2} d x^{2}+d}}-\frac{5 \mathrm{I} b^{2} \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right) \sqrt{c^{2} x^{2}+1}}{3 c^{4} d^{2} \sqrt{c^{2} d x^{2}+d}} \\
& \quad+\frac{5 \mathrm{I} b^{2} \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right) \sqrt{c^{2} x^{2}+1}}{3 c^{4} d^{2} \sqrt{c^{2} d x^{2}+d}}
\end{aligned}
$$

Result(type 4, 704 leaves):
$-\frac{a^{2} x^{2}}{c^{2} d\left(c^{2} d x^{2}+d\right)^{3 / 2}}-\frac{2 a^{2}}{3 d c^{4}\left(c^{2} d x^{2}+d\right)^{3 / 2}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} x^{2}}{\left(c^{2} x^{2}+1\right)^{2} d^{3} c^{2}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x}{3\left(c^{2} x^{2}+1\right)^{3 / 2} d^{3} c^{3}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} x^{2}}{3\left(c^{2} x^{2}+1\right)^{2} d^{3} c^{2}}$

$$
\begin{aligned}
& -\frac{2 b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2}}{3\left(c^{2} x^{2}+1\right)^{2} d^{3} c^{4}}-\frac{b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)}}{3\left(c^{2} x^{2}+1\right)^{2} d^{3} c^{4}}+\frac{5 \mathrm{I} b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \ln \left(1-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{3 \sqrt{c^{2} x^{2}+1} c^{4} d^{3}} \\
& +\frac{5 \mathrm{I} a b \sqrt{d\left(c^{2} x^{2}+1\right)} \ln \left(c x+\sqrt{c^{2} x^{2}+1}+\mathrm{I}\right)}{3 \sqrt{c^{2} x^{2}+1} c^{4} d^{3}}+\frac{5 \mathrm{I} b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{dilog}\left(1-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{3 \sqrt{c^{2} x^{2}+1} c^{4} d^{3}} \\
& -\frac{5 \mathrm{I} b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) \ln \left(1+\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{3 \sqrt{c^{2} x^{2}+1} c^{4} d^{3}}-\frac{2 a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x) x^{2}}{\left(c^{2} x^{2}+1\right)^{2} d^{3} c^{2}}-\frac{a b \sqrt{d\left(c^{2} x^{2}+1\right)} x}{3\left(c^{2} x^{2}+1\right)^{3 / 2} d^{3} c^{3}} \\
& -\frac{4 a b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)}{3\left(c^{2} x^{2}+1\right)^{2} d^{3} c^{4}}-\frac{5 \mathrm{I} a b \sqrt{d\left(c^{2} x^{2}+1\right)} \ln \left(c x+\sqrt{c^{2} x^{2}+1}-\mathrm{I}\right)}{3 \sqrt{c^{2} x^{2}+1} c^{4} d^{3}}-\frac{5 \mathrm{I} b^{2} \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{dilog}\left(1+\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{3 \sqrt{c^{2} x^{2}+1} c^{4} d^{3}}
\end{aligned}
$$

Problem 84: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x\left(c^{2} d x^{2}+d\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 534 leaves, 24 steps):
$\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{3 d\left(c^{2} d x^{2}+d\right)^{3 / 2}}-\frac{b^{2}}{3 d^{2} \sqrt{c^{2} d x^{2}+d}}+\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{d^{2} \sqrt{c^{2} d x^{2}+d}}-\frac{b c x(a+b \operatorname{arcsinh}(c x))}{3 d^{2} \sqrt{c^{2} x^{2}+1} \sqrt{c^{2} d x^{2}+d}}$
$-\underline{14 b(a+b \operatorname{arcsinh}(c x)) \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} x^{2}+1}}-\underline{2(a+b \operatorname{arcsinh}(c x))^{2} \operatorname{arctanh}\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} x^{2}+1}}$

$$
3 d^{2} \sqrt{c^{2} d x^{2}+d} \quad d^{2} \sqrt{c^{2} d x^{2}+d}
$$

$-\frac{2 b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-c x-\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} x^{2}+1}}{d^{2} \sqrt{c^{2} d x^{2}+d}}+\frac{7 \mathrm{I} b^{2} \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right) \sqrt{c^{2} x^{2}+1}}{3 d^{2} \sqrt{c^{2} d x^{2}+d}}$
$-\frac{7 \mathrm{I} b^{2} \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right) \sqrt{c^{2} x^{2}+1}}{3 d^{2} \sqrt{c^{2} d x^{2}+d}}+\frac{2 b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} x^{2}+1}}{d^{2} \sqrt{c^{2} d x^{2}+d}}$
$+\frac{2 b^{2} \text { polylog }\left(3,-c x-\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} x^{2}+1}}{d^{2} \sqrt{c^{2} d x^{2}+d}}-\frac{2 b^{2} \operatorname{polylog}\left(3, c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{c^{2} x^{2}+1}}{d^{2} \sqrt{c^{2} d x^{2}+d}}$
Result(type 8, 28 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{x\left(c^{2} d x^{2}+d\right)^{5 / 2}} \mathrm{~d} x
$$

Problem 87: Unable to integrate problem.

$$
\int \frac{\operatorname{arcsinh}(a x)^{3}}{\left(a^{2} c x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 358 leaves, 18 steps):

$$
\begin{aligned}
& \frac{x \operatorname{arcsinh}(a x)^{3}}{2 c^{2}\left(a^{2} x^{2}+1\right)}-\frac{6 \operatorname{arcsinh}(a x) \arctan \left(a x+\sqrt{a^{2} x^{2}+1}\right)}{a c^{2}}+\frac{\operatorname{arcsinh}(a x)^{3} \arctan \left(a x+\sqrt{a^{2} x^{2}+1}\right)}{a c^{2}}+\frac{3 \mathrm{I} \operatorname{poly} \log \left(2,-\mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{a c^{2}} \\
& \quad-\frac{3 \mathrm{I} \operatorname{arcsinh}(a x)^{2} \operatorname{polylog}\left(2,-\mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{2 a{c^{2}}^{2}}-\frac{3 \mathrm{I} \operatorname{polylog}\left(2, \mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{a c^{2}}+\frac{3 \mathrm{I} \operatorname{arcsinh}(a x)^{2} \operatorname{polylog}\left(2, \mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{2 a c^{2}} \\
& \quad+\frac{3 \mathrm{I} \operatorname{arcsinh}(a x) \operatorname{polylog}\left(3,-\mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{a{c^{2}}^{2}}-\frac{3 \mathrm{I} \operatorname{arcsinh}(a x) \operatorname{polylog}\left(3, \mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{a c^{2}}-\frac{3 \mathrm{I} \operatorname{polylog}\left(4,-\mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{a c^{2}} \\
& \quad+\frac{3 \mathrm{I} \operatorname{polylog}\left(4, \mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{a c^{2}}+\frac{3 \operatorname{arcsinh}(a x)^{2}}{2 a c^{2} \sqrt{a^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\operatorname{arcsinh}(a x)^{3}}{\left(a^{2} c x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Problem 88: Unable to integrate problem.

$$
\int \frac{\operatorname{arcsinh}(a x)^{3}}{\left(a^{2} c x^{2}+c\right)^{3}} d x
$$

Optimal(type 4, 447 leaves, 28 steps):

$$
\begin{aligned}
& -\frac{x \operatorname{arcsinh}(a x)}{4 c^{3}\left(a^{2} x^{2}+1\right)}+\frac{\operatorname{arcsinh}(a x)^{2}}{4 a c^{3}\left(a^{2} x^{2}+1\right)^{3 / 2}}+\frac{x \operatorname{arcsinh}(a x)^{3}}{4 c^{3}\left(a^{2} x^{2}+1\right)^{2}}+\frac{3 x \operatorname{arcsinh}(a x)^{3}}{8 c^{3}\left(a^{2} x^{2}+1\right)}-\frac{5 \operatorname{arcsinh}(a x) \arctan \left(a x+\sqrt{a^{2} x^{2}+1}\right)}{a c^{3}} \\
& \quad+\frac{3 \operatorname{arcsinh}(a x)^{3} \arctan \left(a x+\sqrt{a^{2} x^{2}+1}\right)}{4 a c^{3}}-\frac{9 \mathrm{I} \operatorname{arcsinh}(a x)^{2} \operatorname{polylog}\left(2,-\mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{8 a c^{3}}-\frac{5 \mathrm{I} \operatorname{polylog}\left(2, \mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{2 a c^{3}} \\
& \quad-\frac{9 \mathrm{I} \operatorname{arcsinh}(a x) \operatorname{poly} \log \left(3, \mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{4 a c^{3}}+\frac{9 \mathrm{I} \operatorname{arcsinh}(a x)^{2} \operatorname{polylog}\left(2, \mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{8 a c^{3}} \\
& \quad+\frac{9 \mathrm{I} \operatorname{arcsinh}(a x) \operatorname{polylog}\left(3,-\mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{4 a c^{3}}+\frac{5 \mathrm{I} \operatorname{polylog}\left(2,-\mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{2 a c^{3}}-\frac{9 \mathrm{Ipolylog}\left(4,-\mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{4 a c^{3}} \\
& \quad+\frac{9 \mathrm{I} \operatorname{polylog}\left(4, \mathrm{I}\left(a x+\sqrt{a^{2} x^{2}+1}\right)\right)}{4 a c^{3}}-\frac{1}{4 a c^{3} \sqrt{a^{2} x^{2}+1}}+\frac{9 \operatorname{arcsinh}(a x)^{2}}{8 a c^{3} \sqrt{a^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\operatorname{arcsinh}(a x)^{3}}{\left(a^{2} c x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Problem 112: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c^{2} x^{2}+1\right)^{3 / 2}}{(a+b \operatorname{arcsinh}(c x))^{2}} d x
$$

Optimal(type 4, 145 leaves, 10 steps):

$$
\begin{aligned}
-\frac{\left(c^{2} x^{2}+1\right)^{2}}{b c(a+b \operatorname{arcsinh}(c x))}+\frac{\cosh \left(\frac{2 a}{b}\right) \operatorname{Shi}\left(\frac{2(a+b \operatorname{arcsinh}(c x))}{b}\right)}{b^{2} c}+\frac{\cosh \left(\frac{4 a}{b}\right) \operatorname{Shi}\left(\frac{4(a+b \operatorname{arcsinh}(c x))}{b}\right)}{2 b^{2} c} \\
-\frac{\operatorname{Chi}\left(\frac{2(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{2 a}{b}\right)}{b^{2} c}-\frac{\operatorname{Chi}\left(\frac{4(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{4 a}{b}\right)}{2 b^{2} c}
\end{aligned}
$$

Result(type 4, 419 leaves):

$$
\begin{aligned}
& -\frac{3}{8 c b(a+b \operatorname{arcsinh}(c x))}-\frac{8 c^{4} x^{4}-8 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+8 c^{2} x^{2}-4 \sqrt{c^{2} x^{2}+1} c x+1}{16 c b(a+b \operatorname{arcsinh}(c x))}+\frac{\mathrm{e}^{\frac{4 a}{b}} \mathrm{Ei}_{1}\left(4 \operatorname{arcsinh}(c x)+\frac{4 a}{b}\right)}{4 c b^{2}}-\frac{2 c^{2} x^{2}-2 \sqrt{c^{2} x^{2}+1} c x+1}{4 c b(a+b \operatorname{arcsinh}(c x))} \\
& \quad+\frac{\mathrm{e}^{\frac{2 a}{b}} \operatorname{Ei}_{1}\left(2 \operatorname{arcsinh}(c x)+\frac{2 a}{b}\right)}{2 c b^{2}} \\
& \quad-\frac{2 b c^{2} x^{2}+2 b c \sqrt{c^{2} x^{2}+1} x+2 \operatorname{Ei}_{1}\left(-2 \operatorname{arcsinh}(c x)-\frac{2 a}{b}\right) \mathrm{e}^{-\frac{2 a}{b}} \operatorname{arcsinh}(c x) b+2 \operatorname{Ei}_{1}\left(-2 \operatorname{arcsinh}(c x)-\frac{2 a}{b}\right) \mathrm{e}^{-\frac{2 a}{b}} a+b}{4 c b^{2}(a+b \operatorname{arcsinh}(c x))} \\
& \quad-\frac{1}{16 c b^{2}(a+b \operatorname{arcsinh}(c x))}\left(8 x^{4} b c^{4}+8 \sqrt{c^{2} x^{2}+1} x^{3} b c^{3}+8 b c^{2} x^{2}+4 b c \sqrt{c^{2} x^{2}+1} x+4 \mathrm{e}^{-\frac{4 a}{b}} \operatorname{arcsinh}(c x) \mathrm{Ei}_{1}\left(-4 \operatorname{arcsinh}(c x)-\frac{4 a}{b}\right) b\right. \\
& \left.+4 \mathrm{e}^{-\frac{4 a}{b}} \mathrm{Ei}_{1}\left(-4 \operatorname{arcsinh}(c x)-\frac{4 a}{b}\right) a+b\right)
\end{aligned}
$$

Problem 114: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(c^{2} x^{2}+1\right)^{5 / 2}}{(a+b \operatorname{arcsinh}(c x))^{2}} \mathrm{~d} x
$$

Optimal(type 4, 204 leaves, 13 steps):

$$
\begin{aligned}
-\frac{\left(c^{2} x^{2}+1\right)^{3}}{b c(a+b \operatorname{arcsinh}(c x))}+\frac{15 \cosh \left(\frac{2 a}{b}\right) \operatorname{Shi}\left(\frac{2(a+b \operatorname{arcsinh}(c x))}{b}\right)}{16 b^{2} c}+\frac{3 \cosh \left(\frac{4 a}{b}\right) \operatorname{Shi}\left(\frac{4(a+b \operatorname{arcsinh}(c x))}{b}\right)}{4 b^{2} c} \\
+\frac{3 \cosh \left(\frac{6 a}{b}\right) \operatorname{Shi}\left(\frac{6(a+b \operatorname{arcsinh}(c x))}{b}\right)}{16 b^{2} c}-\frac{15 \operatorname{Chi}\left(\frac{2(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{2 a}{b}\right)}{16 b^{2} c}-\frac{3 \operatorname{Chi}\left(\frac{4(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{4 a}{b}\right)}{4 b^{2} c}
\end{aligned}
$$

$$
-\frac{3 \operatorname{Chi}\left(\frac{6(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{6 a}{b}\right)}{16 b^{2} c}
$$

Result(type 4, 703 leaves):
$-\frac{5}{16 c b(a+b \operatorname{arcsinh}(c x))}-\frac{32 x^{6} c^{6}-32 \sqrt{c^{2} x^{2}+1} x^{5} c^{5}+48 c^{4} x^{4}-32 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+18 c^{2} x^{2}-6 \sqrt{c^{2} x^{2}+1} c x+1}{64 c b(a+b \operatorname{arcsinh}(c x))}$

$$
\begin{aligned}
& +\frac{3 \mathrm{e}^{\frac{6 a}{b}} \operatorname{Ei}_{1}\left(6 \operatorname{arcsinh}(c x)+\frac{6 a}{b}\right)}{32 c b^{2}}-\frac{3\left(8 c^{4} x^{4}-8 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+8 c^{2} x^{2}-4 \sqrt{c^{2} x^{2}+1} c x+1\right)}{32 c b(a+b \operatorname{arcsinh}(c x))}+\frac{3 \mathrm{e}^{\frac{4 a}{b}} \operatorname{Ei_{1}}\left(4 \operatorname{arcsinh}(c x)+\frac{4 a}{b}\right)}{8 c b^{2}} \\
& -\frac{15\left(2 c^{2} x^{2}-2 \sqrt{c^{2} x^{2}+1} c x+1\right)}{64 c b(a+b \operatorname{arcsinh}(c x))}+\frac{15 \mathrm{e}^{\frac{2 a}{b}} \operatorname{Ei}_{1}\left(2 \operatorname{arcsinh}(c x)+\frac{2 a}{b}\right)}{32 c b^{2}} \\
& -\frac{15\left(2 b c^{2} x^{2}+2 b c \sqrt{c^{2} x^{2}+1} x+2 \operatorname{Ei}_{1}\left(-2 \operatorname{arcsinh}(c x)-\frac{2 a}{b}\right) \mathrm{e}^{-\frac{2 a}{b}} \operatorname{arcsinh}(c x) b+2 \operatorname{Ei}_{1}\left(-2 \operatorname{arcsinh}(c x)-\frac{2 a}{b}\right) \mathrm{e}^{-\frac{2 a}{b}} a+b\right)}{64 c b^{2}(a+b \operatorname{arcsinh}(c x))} \\
& -\frac{1}{32 c b^{2}(a+b \operatorname{arcsinh}(c x))}\left(3 \left(8 x^{4} b c^{4}+8 \sqrt{c^{2} x^{2}+1} x^{3} b c^{3}+8 b c^{2} x^{2}+4 b c \sqrt{c^{2} x^{2}+1} x+4 \mathrm{e}^{-\frac{4 a}{b}} \operatorname{arcsinh}(c x) \mathrm{Ei}_{1}\left(-4 \operatorname{arcsinh}(c x)-\frac{4 a}{b}\right) b\right.\right. \\
& \left.\left.+4 \mathrm{e}^{-\frac{4 a}{b}} \operatorname{Ei}_{1}\left(-4 \operatorname{arcsinh}(c x)-\frac{4 a}{b}\right) a+b\right)\right)-\frac{1}{64 c b^{2}(a+b \operatorname{arcsinh}(c x))}\left(32 x^{6} b c^{6}+32 \sqrt{c^{2} x^{2}+1} x^{5} b c^{5}+48 x^{4} b c^{4}+32 \sqrt{c^{2} x^{2}+1} x^{3} b c^{3}\right. \\
& \left.+18 b c^{2} x^{2}+6 b c \sqrt{c^{2} x^{2}+1} x+6 \operatorname{arcsinh}(c x) \operatorname{Ei}_{1}\left(-6 \operatorname{arcsinh}(c x)-\frac{6 a}{b}\right) \mathrm{e}^{-\frac{6 a}{b}} b+6 \mathrm{Ei}_{1}\left(-6 \operatorname{arcsinh}(c x)-\frac{6 a}{b}\right) \mathrm{e}^{-\frac{6 a}{b}} a+b\right)
\end{aligned}
$$

Problem 117: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}}{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 137 leaves, 10 steps):

$$
-\frac{x^{4}}{b c(a+b \operatorname{arcsinh}(c x))}-\frac{\cosh \left(\frac{2 a}{b}\right) \operatorname{Shi}\left(\frac{2(a+b \operatorname{arcsinh}(c x))}{b}\right)}{b^{2} c^{5}}+\frac{\cosh \left(\frac{4 a}{b}\right) \operatorname{Shi}\left(\frac{4(a+b \operatorname{arcsinh}(c x))}{b}\right)}{2 b^{2} c^{5}}
$$

$$
+\frac{\operatorname{Chi}\left(\frac{2(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{2 a}{b}\right)}{b^{2} c^{5}}-\frac{\operatorname{Chi}\left(\frac{4(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{4 a}{b}\right)}{2 b^{2} c^{5}}
$$

Result(type 4, 419 leaves):

$$
\begin{aligned}
& -\frac{3}{8 c^{5} b(a+b \operatorname{arcsinh}(c x))}-\frac{8 c^{4} x^{4}-8 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+8 c^{2} x^{2}-4 \sqrt{c^{2} x^{2}+1} c x+1}{16 c^{5} b(a+b \operatorname{arcsinh}(c x))}+\frac{\mathrm{e}^{\frac{4 a}{b}} \operatorname{Ei}_{1}\left(4 \operatorname{arcsinh}(c x)+\frac{4 a}{b}\right)}{4 c^{5} b^{2}} \\
& \quad+\frac{2 c^{2} x^{2}-2 \sqrt{c^{2} x^{2}+1} c x+1}{4 c^{5} b(a+b \operatorname{arcsinh}(c x))}-\frac{\mathrm{e}^{\frac{2 a}{b}} \operatorname{Ei}_{1}\left(2 \operatorname{arcsinh}(c x)+\frac{2 a}{b}\right)}{2 c^{5} b^{2}} \\
& \quad+\frac{2 b c^{2} x^{2}+2 b c \sqrt{c^{2} x^{2}+1} x+2 \operatorname{Ei}_{1}\left(-2 \operatorname{arcsinh}(c x)-\frac{2 a}{b}\right) \mathrm{e}^{-\frac{2 a}{b}} \operatorname{arcsinh}(c x) b+2 \operatorname{Ei}_{1}\left(-2 \operatorname{arcsinh}(c x)-\frac{2 a}{b}\right) \mathrm{e}^{-\frac{2 a}{b}} a+b}{4 c^{5} b^{2}(a+b \operatorname{arcsinh}(c x))} \\
& \quad-\frac{1}{16 c^{5} b^{2}(a+b \operatorname{arcsinh}(c x))}\left(8 x^{4} b c^{4}+8 \sqrt{c^{2} x^{2}+1} x^{3} b c^{3}+8 b c^{2} x^{2}+4 b c \sqrt{c^{2} x^{2}+1} x+4 \mathrm{e}^{-\frac{4 a}{b}} \operatorname{arcsinh}(c x) \mathrm{Ei}_{1}\left(-4 \operatorname{arcsinh}(c x)-\frac{4 a}{b}\right) b\right. \\
& \left.\quad+4 \mathrm{e}^{-\frac{4 a}{b}} \mathrm{Ei}_{1}\left(-4 \operatorname{arcsinh}(c x)-\frac{4 a}{b}\right) a+b\right)
\end{aligned}
$$

Problem 118: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}}{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 134 leaves, 10 steps):
$-\frac{x^{3}}{b c(a+b \operatorname{arcsinh}(c x))}-\frac{3 \operatorname{Chi}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right) \cosh \left(\frac{a}{b}\right)}{4 b^{2} c^{4}}+\frac{3 \operatorname{Chi}\left(\frac{3(a+b \operatorname{arcsinh}(c x))}{b}\right) \cosh \left(\frac{3 a}{b}\right)}{4 b^{2} c^{4}}+\frac{3 \operatorname{Shi}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right) \sinh \left(\frac{a}{b}\right)}{4 b^{2} c^{4}}$

$$
-\frac{3 \operatorname{Shi}\left(\frac{3(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{3 a}{b}\right)}{4 b^{2} c^{4}}
$$

Result(type 4, 363 leaves):

$$
-\frac{4 c^{3} x^{3}-4 c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+3 c x-\sqrt{c^{2} x^{2}+1}}{8 c^{4} b(a+b \operatorname{arcsinh}(c x))}-\frac{3 \mathrm{e}^{\frac{3 a}{b}} \mathrm{Ei}_{1}\left(3 \operatorname{arcsinh}(c x)+\frac{3 a}{b}\right)}{8 c^{4} b^{2}}+\frac{3\left(c x-\sqrt{c^{2} x^{2}+1}\right)}{8 c^{4} b(a+b \operatorname{arcsinh}(c x))}+\frac{3 \mathrm{e}^{\frac{a}{b}} \operatorname{Ei}_{1}\left(\operatorname{arcsinh}(c x)+\frac{a}{b}\right)}{8 c^{4} b^{2}}
$$

$$
+\frac{3\left(\mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(c x)-\frac{a}{b}\right) \operatorname{arcsinh}(c x) b+\mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(c x)-\frac{a}{b}\right) a+x b c+\sqrt{c^{2} x^{2}+1} b\right)}{04 h^{2}}
$$

$$
8 c^{4} b^{2}(a+b \operatorname{arcsinh}(c x))
$$

$$
-\frac{4 x^{3} b c^{3}+4 \sqrt{c^{2} x^{2}+1} x^{2} b c^{2}+3 \mathrm{e}^{-\frac{3 a}{b}} \operatorname{arcsinh}(c x) \operatorname{Ei}_{1}\left(-3 \operatorname{arcsinh}(c x)-\frac{3 a}{b}\right) b+3 \mathrm{e}^{-\frac{3 a}{b}} \operatorname{Ei}_{1}\left(-3 \operatorname{arcsinh}(c x)-\frac{3 a}{b}\right) a+3 x b c+\sqrt{c^{2} x^{2}+1} b}{}
$$

Problem 119: Result more than twice size of optimal antiderivative.

$$
\int \frac{x}{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 73 leaves, 5 steps):


Result(type 4, 150 leaves):

$$
\begin{aligned}
& -\frac{c x-\sqrt{c^{2} x^{2}+1}}{2 c^{2} b(a+b \operatorname{arcsinh}(c x))}-\frac{\mathrm{e}^{\frac{a}{b}} \operatorname{Ei}_{1}\left(\operatorname{arcsinh}(c x)+\frac{a}{b}\right)}{2 c^{2} b^{2}} \\
& -\frac{\mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(c x)-\frac{a}{b}\right) \operatorname{arcsinh}(c x) b+\mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(c x)-\frac{a}{b}\right) a+x b c+\sqrt{c^{2} x^{2}+1} b}{2 c^{2} b^{2}(a+b \operatorname{arcsinh}(c x))}
\end{aligned}
$$

Problem 125: Unable to integrate problem.

$$
\int \frac{x^{3}\left(c^{2} d x^{2}+d\right)}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 198 leaves, 27 steps):


$$
+\frac{d \operatorname{erfi}\left(\frac{\sqrt{6} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{6} \sqrt{\pi}}{32 b^{3 / 2} c^{4} \mathrm{e}^{\frac{6 a}{b}}}-\frac{2 d x^{3}\left(c^{2} x^{2}+1\right)^{3 / 2}}{b c \sqrt{a+b \operatorname{arcsinh}(c x)}}
$$

Result(type 8, 26 leaves):

$$
\int \frac{x^{3}\left(c^{2} d x^{2}+d\right)}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Problem 126: Unable to integrate problem.

$$
\int \frac{x^{2}\left(c^{2} d x^{2}+d\right)}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 266 leaves, 32 steps):
$\frac{d \mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{8 b^{3 / 2} c^{3}}-\frac{d \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{8 b^{3 / 2} c^{3} \mathrm{e}^{\frac{a}{b}}}-\frac{d \mathrm{e}^{\frac{3 a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{16 b^{3 / 2} c^{3}}$
$+\frac{d \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{16 b^{3 / 2} c^{3} \mathrm{e}^{\frac{3 a}{b}}}-\frac{d \mathrm{e}^{\frac{5 a}{b}} \operatorname{erf}\left(\frac{\sqrt{5} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{5} \sqrt{\pi}}{16 b^{3 / 2} c^{3}}+\frac{d \operatorname{erfi}\left(\frac{\sqrt{5} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{5} \sqrt{\pi}}{16 b^{3 / 2} c^{3} \mathrm{e}^{\frac{5 a}{b}}}$

$$
-\frac{2 d x^{2}\left(c^{2} x^{2}+1\right)^{3 / 2}}{b c \sqrt{a+b \operatorname{arcsinh}(c x)}}
$$

Result (type 8, 26 leaves):

$$
\int \frac{x^{2}\left(c^{2} d x^{2}+d\right)}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Problem 127: Unable to integrate problem.

$$
\int \frac{x^{3}\left(c^{2} d x^{2}+d\right)^{2}}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 372 leaves, 32 steps):

$+\frac{d^{2} \mathrm{e}^{\frac{6 a}{b}} \operatorname{erf}\left(\frac{\sqrt{6} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{6} \sqrt{\pi}}{64 b^{3 / 2} c^{4}}+\frac{d^{2} \operatorname{erfi}\left(\frac{\sqrt{6} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{6} \sqrt{\pi}}{64 b^{3 / 2} c^{4} \mathrm{e}^{\frac{6 a}{b}}}-\frac{2 d^{2} x^{3}\left(c^{2} x^{2}+1\right)^{5 / 2}}{b c \sqrt{a+b \operatorname{arcsinh}(c x)}}$
Result(type 8, 28 leaves):

$$
\int \frac{x^{3}\left(c^{2} d x^{2}+d\right)^{2}}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Problem 128: Unable to integrate problem.

$$
\int \frac{x^{2}\left(c^{2} d x^{2}+d\right)^{2}}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 366 leaves, 42 steps):
$\frac{5 d^{2} \mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{64 b^{3 / 2} c^{3}}-\frac{5 d^{2} \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{64 b^{3} / 2 c^{3} \mathrm{e}^{\frac{a}{b}}}-\frac{d^{2} \mathrm{e}^{\frac{3 a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{64 b^{3 / 2} c^{3}}$

$-\frac{d^{2} \mathrm{e}^{\frac{7 a}{b}} \operatorname{erf}\left(\frac{\sqrt{7} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{7} \sqrt{\pi}}{64 b^{3 / 2} c^{3}}+\frac{d^{2} \operatorname{erfi}\left(\frac{\sqrt{7} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{7} \sqrt{\pi}}{64 b^{3 / 2} c^{3} \mathrm{e}^{\frac{7 a}{b}}}-\frac{2 d^{2} x^{2}\left(c^{2} x^{2}+1\right)^{5 / 2}}{b c \sqrt{a+b \operatorname{arcsinh}(c x)}}$
Result(type 8, 28 leaves):

$$
\int \frac{x^{2}\left(c^{2} d x^{2}+d\right)^{2}}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Problem 130: Unable to integrate problem.

$$
\int\left(a^{2} c x^{2}+c\right)^{3 / 2} \sqrt{\operatorname{arcsinh}(a x)} \mathrm{d} x
$$

Optimal(type 4, 253 leaves, 24 steps):


$+\frac{c \operatorname{erf}(2 \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi} \sqrt{a^{2} c x^{2}+c}}{256 a \sqrt{a^{2} x^{2}+1}}-\frac{c \operatorname{erfi}(2 \sqrt{\operatorname{arcsinh}(a x)}) \sqrt{\pi} \sqrt{a^{2} c x^{2}+c}}{256 a \sqrt{a^{2} x^{2}+1}}+\frac{x\left(a^{2} c x^{2}+c\right)^{3 / 2} \sqrt{\operatorname{arcsinh}(a x)}}{4}$
$+\frac{3 c x \sqrt{a^{2} c x^{2}+c} \sqrt{\operatorname{arcsinh}(a x)}}{8}$
Result(type 8, 21 leaves):

$$
\int\left(a^{2} c x^{2}+c\right)^{3 / 2} \sqrt{\operatorname{arcsinh}(a x)} \mathrm{d} x
$$

Problem 134: Unable to integrate problem.

$$
\int \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3 / 2} \sqrt{a^{2}+x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 203 leaves, 11 steps):
$\frac{x \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3 / 2} \sqrt{a^{2}+x^{2}}}{2}+\frac{a \operatorname{arcsinh}\left(\frac{x}{a}\right)^{5 / 2} \sqrt{a^{2}+x^{2}}}{5 \sqrt{1+\frac{x^{2}}{a^{2}}}}+\frac{3 a \operatorname{erf}\left(\sqrt{2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right) \sqrt{2} \sqrt{\pi} \sqrt{a^{2}+x^{2}}}{128 \sqrt{1+\frac{x^{2}}{a^{2}}}}$

$$
+\frac{3 a \operatorname{erfi}\left(\sqrt{2} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\right) \sqrt{2} \sqrt{\pi} \sqrt{a^{2}+x^{2}}}{128 \sqrt{1+\frac{x^{2}}{a^{2}}}}-\frac{3 a \sqrt{a^{2}+x^{2}} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{16 \sqrt{1+\frac{x^{2}}{a^{2}}}}-\frac{3 x^{2} \sqrt{a^{2}+x^{2}} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{8 a \sqrt{1+\frac{x^{2}}{a^{2}}}}
$$

Result(type 8, 20 leaves):

$$
\int \operatorname{arcsinh}\left(\frac{x}{a}\right)^{3 / 2} \sqrt{a^{2}+x^{2}} \mathrm{~d} x
$$

Problem 136: Unable to integrate problem.

$$
\int \frac{x}{\sqrt{x^{2}+1} \sqrt{\operatorname{arcsinh}(x)}} \mathrm{d} x
$$

Optimal(type 4, 21 leaves, 6 steps):

$$
-\frac{\operatorname{erf}(\sqrt{\operatorname{arcsinh}(x)}) \sqrt{\pi}}{2}+\frac{\operatorname{erfi}(\sqrt{\operatorname{arcsinh}(x)}) \sqrt{\pi}}{2}
$$

Result(type 8, 15 leaves):

$$
\int \frac{x}{\sqrt{x^{2}+1} \sqrt{\operatorname{arcsinh}(x)}} \mathrm{d} x
$$

Problem 140: Unable to integrate problem.

$$
\int x\left(c^{2} d x^{2}+d\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{n} \mathrm{~d} x
$$

Optimal(type 4, 706 leaves, 15 steps):
$\underline{7^{-1-n} d^{2}(a+b \operatorname{arcsinh}(c x))^{n} \Gamma\left(1+n,-\frac{7(a+b \operatorname{arcsinh}(c x))}{b}\right) \sqrt{c^{2} d x^{2}+d}}$

$$
128 c^{2} \mathrm{e}^{\frac{7 a}{b}}\left(\frac{-a-b \operatorname{arcsinh}(c x)}{b}\right)^{n} \sqrt{c^{2} x^{2}+1}
$$

$$
\begin{aligned}
& +\frac{d^{2}(a+b \operatorname{arcsinh}(c x))^{n} \Gamma\left(1+n,-\frac{5(a+b \operatorname{arcsinh}(c x))}{b}\right) \sqrt{c^{2} d x^{2}+d}}{1285^{n} c^{2} \mathrm{e}^{\frac{5 a}{b}}\left(\frac{-a-b \operatorname{arcsinh}(c x)}{b}\right)^{n} \sqrt{c^{2} x^{2}+1}} \\
& +\frac{3^{1-n} d^{2}(a+b \operatorname{arcsinh}(c x))^{n} \Gamma\left(1+n,-\frac{3(a+b \operatorname{arcsinh}(c x))}{b}\right) \sqrt{c^{2} d x^{2}+d}}{} \\
& 128 c^{2} \mathrm{e}^{\frac{3 a}{b}}\left(\frac{-a-b \operatorname{arcsinh}(c x)}{b}\right)^{n} \sqrt{c^{2} x^{2}+1} \\
& +\frac{5 d^{2}(a+b \operatorname{arcsinh}(c x))^{n} \Gamma\left(1+n, \frac{-a-b \operatorname{arcsinh}(c x)}{b}\right) \sqrt{c^{2} d x^{2}+d}}{128 c^{2} \mathrm{e}^{\frac{a}{b}}\left(\frac{-a-b \operatorname{arcsinh}(c x)}{b}\right)^{n} \sqrt{c^{2} x^{2}+1}}+\frac{5 d^{2} \mathrm{e}^{\frac{a}{b}}(a+b \operatorname{arcsinh}(c x))^{n} \Gamma\left(1+n, \frac{a+b \operatorname{arcsinh}(c x)}{b}\right) \sqrt{c^{2} d x^{2}+d}}{128 c^{2}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right)^{n} \sqrt{c^{2} x^{2}+1}} \\
& +\frac{3^{1-n} d^{2} \mathrm{e}^{\frac{3 a}{b}}(a+b \operatorname{arcsinh}(c x))^{n} \Gamma\left(1+n, \frac{3(a+b \operatorname{arcsinh}(c x))}{b}\right) \sqrt{c^{2} d x^{2}+d}}{\text { ( }} \\
& 128 c^{2}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right)^{n} \sqrt{c^{2} x^{2}+1} \\
& +\frac{d^{2} \mathrm{e}^{\frac{5 a}{b}}(a+b \operatorname{arcsinh}(c x))^{n} \Gamma\left(1+n, \frac{5(a+b \operatorname{arcsinh}(c x))}{b}\right) \sqrt{c^{2} d x^{2}+d}}{n} \\
& 1285^{n} c^{2}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right)^{n} \sqrt{c^{2} x^{2}+1} \\
& +\frac{7^{-1-n} d^{2} \mathrm{e}^{\frac{7 a}{b}}(a+b \operatorname{arcsinh}(c x))^{n} \Gamma\left(1+n, \frac{7(a+b \operatorname{arcsinh}(c x))}{b}\right) \sqrt{c^{2} d x^{2}+d}}{128 c^{2}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right)^{n} \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

Result(type 8, 26 leaves):

$$
\int x\left(c^{2} d x^{2}+d\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{n} \mathrm{~d} x
$$

Problem 141: Unable to integrate problem.

$$
\int \frac{x^{3} \operatorname{arcsinh}(a x)^{n}}{\sqrt{a^{2} x^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 105 leaves, 9 steps):
$\frac{3^{-1-n} \operatorname{arcsinh}(a x)^{n} \Gamma(1+n,-3 \operatorname{arcsinh}(a x))}{8 a^{4}(-\operatorname{arcsinh}(a x))^{n}}-\frac{3 \operatorname{arcsinh}(a x)^{n} \Gamma(1+n,-\operatorname{arcsinh}(a x))}{8 a^{4}(-\operatorname{arcsinh}(a x))^{n}}-\frac{3 \Gamma(1+n, \operatorname{arcsinh}(a x))}{8 a^{4}}+\frac{3^{-1-n} \Gamma(1+n, 3 \operatorname{arcsinh}(a x))}{8 a^{4}}$
Result(type 8, 23 leaves):

$$
\int \frac{x^{3} \operatorname{arcsinh}(a x)^{n}}{\sqrt{a^{2} x^{2}+1}} \mathrm{~d} x
$$

Problem 142: Unable to integrate problem.

$$
\int \frac{x \operatorname{arcsinh}(a x)^{n}}{\sqrt{a^{2} x^{2}+1}} \mathrm{~d} x
$$

Optimal(type 4, 45 leaves, 4 steps):

$$
\frac{\operatorname{arcsinh}(a x)^{n} \Gamma(1+n,-\operatorname{arcsinh}(a x))}{2 a^{2}(-\operatorname{arcsinh}(a x))^{n}}+\frac{\Gamma(1+n, \operatorname{arcsinh}(a x))}{2 a^{2}}
$$

Result(type 8, 21 leaves):

$$
\int \frac{x \operatorname{arcsinh}(a x)^{n}}{\sqrt{a^{2} x^{2}+1}} \mathrm{~d} x
$$

Problem 143: Unable to integrate problem.

$$
\int(a+b \operatorname{arcsinh}(c x)) \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x} \mathrm{~d} x
$$

Optimal(type 3, 119 leaves, 4 steps):

$$
\frac{x(a+b \operatorname{arcsinh}(c x)) \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}{2}-\frac{b c x^{2} \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}{4 \sqrt{c^{2} x^{2}+1}}+\frac{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}{4 b c \sqrt{c^{2} x^{2}+1}}
$$

Result(type 8, 31 leaves):

$$
\int(a+b \operatorname{arcsinh}(c x)) \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x} \mathrm{~d} x
$$

Problem 144: Unable to integrate problem.

$$
\int(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x)) \mathrm{d} x
$$

Optimal(type 3, 371 leaves, 12 steps):

$$
\begin{aligned}
& -\frac{\mathrm{I} b d x(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{5\left(c^{2} x^{2}+1\right)^{3 / 2}}-\frac{5 b c d x^{2}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{16\left(c^{2} x^{2}+1\right)^{3 / 2}}-\frac{2 \mathrm{I} b c^{2} d x^{3}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{15\left(c^{2} x^{2}+1\right)^{3 / 2}} \\
& -\frac{b c^{3} d x^{4}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{16\left(c^{2} x^{2}+1\right)^{3 / 2}}-\frac{\mathrm{I} b c^{4} d x^{5}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{25\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{d x(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{4} \\
& \quad+\frac{3 d x(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{8\left(c^{2} x^{2}+1\right)}+\frac{\mathrm{I} d(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{5 c}
\end{aligned}
$$

$$
+\frac{3 d(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{16 b c\left(c^{2} x^{2}+1\right)^{3 / 2}}
$$

Result(type 8, 31 leaves):

$$
\int(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x)) \mathrm{d} x
$$

Problem 145: Unable to integrate problem.

$$
\int(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x)) \mathrm{d} x
$$

Optimal(type 3, 201 leaves, 7 steps):

$$
\begin{array}{r}
-\frac{5 b c x^{2}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{16\left(c^{2} x^{2}+1\right)^{3 / 2}}-\frac{b c^{3} x^{4}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{16\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{x(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{4} \\
+\frac{3 x(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{8\left(c^{2} x^{2}+1\right)}+\frac{3(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{16 b c\left(c^{2} x^{2}+1\right)^{3 / 2}}
\end{array}
$$

Result(type 8, 31 leaves):

$$
\int(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x)) \mathrm{d} x
$$

Problem 146: Unable to integrate problem.

$$
\int(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \mathrm{d} x
$$

Optimal(type 3, 371 leaves, 12 steps):
$\frac{\mathrm{I} b f x(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{5\left(c^{2} x^{2}+1\right)^{3 / 2}}-\frac{5 b c f x^{2}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{16\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{2 \mathrm{I} b c^{2} f x^{3}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{15\left(c^{2} x^{2}+1\right)^{3 / 2}}$
$-\frac{b c^{3} f x^{4}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{16\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{\mathrm{I} b c^{4} f x^{5}(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}{25\left(c^{2} x^{2}+1\right)^{3 / 2}}+\frac{f x(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{4}$
$+\frac{3 f x(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{8\left(c^{2} x^{2}+1\right)}-\frac{\mathrm{I} f(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{5 c}$
$+\frac{3 f(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{16 b c\left(c^{2} x^{2}+1\right)^{3 / 2}}$
Result(type 8, 31 leaves):

$$
\int(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \mathrm{d} x
$$

[^0]$$
\int \frac{(d+\mathrm{I} c d x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{\sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Optimal(type 3, 222 leaves, 9 steps):
$\frac{2 \mathrm{I} d^{2}\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}-\frac{d^{2} x\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{2 \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}-\frac{2 \mathrm{I} b d^{2} x \sqrt{c^{2} x^{2}+1}}{\sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}+\frac{b c d^{2} x^{2} \sqrt{c^{2} x^{2}+1}}{4 \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}$

$$
+\frac{3 d^{2}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} x^{2}+1}}{4 b c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}
$$

Result(type 8, 31 leaves):

$$
\int \frac{(d+\mathrm{I} c d x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{\sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Problem 148: Unable to integrate problem.

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(d+\mathrm{I} c d x)^{3 / 2} \sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Optimal(type 3, 95 leaves, 5 steps):

$$
\frac{f(\mathrm{I}+c x)\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}-\frac{b f\left(c^{2} x^{2}+1\right)^{3 / 2} \ln (\mathrm{I}-c x)}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}
$$

Result(type 8, 31 leaves):

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(d+\mathrm{I} c d x)^{3 / 2} \sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Problem 149: Unable to integrate problem.

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(d+\mathrm{I} c d x)^{5 / 2} \sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Optimal(type 3, 244 leaves, 8 steps):

$$
\begin{aligned}
& \frac{\mathrm{I} b f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2}}{3 c(\mathrm{I}-c x)(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{2 \mathrm{I} f^{2}(1-\mathrm{I} c x)\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{f^{2} x\left(c^{2} x^{2}+1\right)^{2}(a+b \operatorname{arcsinh}(c x))}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& -\frac{\mathrm{I} b f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \arctan (c x)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{b f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \ln \left(c^{2} x^{2}+1\right)}{6 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}
\end{aligned}
$$

Result(type 8, 31 leaves):

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(d+\mathrm{I} c d x)^{5 / 2} \sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Problem 150: Unable to integrate problem.

$$
\int \frac{(d+\mathrm{I} c d x)^{5 / 2}(a+b \operatorname{arcsinh}(c x))}{(f-\mathrm{I} c f x)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 431 leaves, 7 steps):

$$
\begin{aligned}
& \frac{3 \mathrm{I} b d^{4} x\left(c^{2} x^{2}+1\right)^{3 / 2}}{2(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}+\frac{b c d^{4} x^{2}\left(c^{2} x^{2}+1\right)^{3 / 2}}{(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}+\frac{5 b d^{4}(1+\mathrm{I} c x)^{2}\left(c^{2} x^{2}+1\right)^{3 / 2}}{4 c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}+\frac{15 b d^{4}\left(c^{2} x^{2}+1\right)^{3 / 2} \operatorname{arcsinh}(c x)^{2}}{4 c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}} \\
& -\frac{2 \mathrm{I} d^{4}(1+\mathrm{I} c x)^{3}\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}-\frac{15 \mathrm{I} d^{4}\left(c^{2} x^{2}+1\right)^{2}(a+b \operatorname{arcsinh}(c x))}{2 c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}-\frac{5 \mathrm{I} d^{4}(1+\mathrm{I} c x)\left(c^{2} x^{2}+1\right)^{2}(a+b \operatorname{arcsinh}(c x))}{2 c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}} \\
& \quad-\frac{15 d^{4}\left(c^{2} x^{2}+1\right)^{3 / 2} \operatorname{arcsinh}(c x)(a+b \operatorname{arcsinh}(c x))}{2 c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}-\frac{8 b d^{4}\left(c^{2} x^{2}+1\right)^{3 / 2} \ln (\mathrm{I}+c x)}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}
\end{aligned}
$$

Result(type 8, 31 leaves):

$$
\int \frac{(d+\mathrm{I} c d x)^{5 / 2}(a+b \operatorname{arcsinh}(c x))}{(f-\mathrm{I} c f x)^{3 / 2}} \mathrm{~d} x
$$

Problem 151: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x)) \sqrt{d+\mathrm{I} c d x}}{(f-\mathrm{I} c f x)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 153 leaves, 8 steps):

$$
-\frac{2 \mathrm{I} d^{2}(1+\mathrm{I} c x)\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}-\frac{d^{2}\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{2 b c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}-\frac{2 b d^{2}\left(c^{2} x^{2}+1\right)^{3 / 2} \ln (\mathrm{I}+c x)}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}
$$

Result(type 8, 31 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x)) \sqrt{d+\mathrm{I} c d x}}{(f-\mathrm{I} c f x)^{3 / 2}} \mathrm{~d} x
$$

Problem 152: Unable to integrate problem.

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(f-\mathrm{I} c f x)^{3 / 2} \sqrt{d+\mathrm{I} c d x}} \mathrm{~d} x
$$

Optimal(type 3, 96 leaves, 5 steps):

$$
-\frac{d(\mathrm{I}-c x)\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}-\frac{b d\left(c^{2} x^{2}+1\right)^{3 / 2} \ln (\mathrm{I}+c x)}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}
$$

Result(type 8, 31 leaves):

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(f-\mathrm{I} c f x)^{3} / 2 \sqrt{d+\mathrm{I} c d x}} \mathrm{~d} x
$$

Problem 153: Unable to integrate problem.

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(f-\mathrm{I} c f x)^{5 / 2} \sqrt{d+\mathrm{I} c d x}} \mathrm{~d} x
$$

Optimal(type 3, 243 leaves, 8 steps):

$$
\begin{aligned}
& \frac{\mathrm{I} b d^{2}\left(c^{2} x^{2}+1\right)^{5 / 2}}{3 c(\mathrm{I}+c x)(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{2 \mathrm{I} d^{2}(1+\mathrm{I} c x)\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{d^{2} x\left(c^{2} x^{2}+1\right)^{2}(a+b \operatorname{arcsinh}(c x))}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& \quad+\frac{\mathrm{I} b d^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \arctan (c x)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{b d^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \ln \left(c^{2} x^{2}+1\right)}{6 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}
\end{aligned}
$$

Result(type 8, 31 leaves):

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(f-\mathrm{I} c f x)^{5} / 2 \sqrt{d+\mathrm{I} c d x}} \mathrm{~d} x
$$

Problem 154: Unable to integrate problem.

$$
\int(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x} \mathrm{~d} x
$$

Optimal(type 3, 198 leaves, 6 steps):
$\frac{b^{2} x \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}{4}+\frac{x(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}{2}-\frac{b^{2} \operatorname{arcsinh}(c x) \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}{4 c \sqrt{c^{2} x^{2}+1}}$
$-\frac{b c x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}{\sqrt{2}}+\frac{(a+b \operatorname{arcsinh}(c x))^{3} \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}{}$

$$
2 \sqrt{c^{2} x^{2}+1} \quad 6 b c \sqrt{c^{2} x^{2}+1}
$$

Result(type 8, 33 leaves):

$$
\int(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x} \mathrm{~d} x
$$

Problem 155: Unable to integrate problem.

$$
\int \frac{(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{\sqrt{d+\mathrm{I} c d x}} \mathrm{~d} x
$$

Optimal(type 3, 367 leaves, 11 steps):

$$
\begin{aligned}
& -\frac{4 \mathrm{I} b^{2} f^{2}\left(c^{2} x^{2}+1\right)}{c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}-\frac{b^{2} f^{2} x\left(c^{2} x^{2}+1\right)}{4 \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}-\frac{2 \mathrm{I} f^{2}\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}-\frac{f^{2} x\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{2 \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}} \\
& \quad+\frac{b^{2} f^{2} \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1}}{4 c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}+\frac{4 \mathrm{I} b f^{2} x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{\sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}+\frac{b c f^{2} x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{2 \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}} \\
& \quad+\frac{f^{2}(a+b \operatorname{arcsinh}(c x))^{3} \sqrt{c^{2} x^{2}+1}}{2 b c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}
\end{aligned}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(f-\mathrm{I} c f x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{\sqrt{d+\mathrm{I} c d x}} \mathrm{~d} x
$$

Problem 156: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{d+\mathrm{I} c d x}}{\sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Optimal(type 3, 217 leaves, 8 steps):

$$
\begin{aligned}
& \frac{2 \mathrm{I} b^{2} d\left(c^{2} x^{2}+1\right)}{c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}+\frac{\mathrm{I} d\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}-\frac{2 \mathrm{I} a b d x \sqrt{c^{2} x^{2}+1}}{\sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}-\frac{2 \mathrm{I} b^{2} d x \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1}}{\sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}} \\
& \quad+\frac{d(a+b \operatorname{arcsinh}(c x))^{3} \sqrt{c^{2} x^{2}+1}}{3 b c \sqrt{d+\mathrm{I} c d x} \sqrt{f-\mathrm{I} c f x}}
\end{aligned}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{d+\mathrm{I} c d x}}{\sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Problem 157: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{5} / 2 \sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Optimal(type 4, 833 leaves, 30 steps):

$$
\begin{aligned}
& -\frac{2 \mathrm{I} b^{2} f^{2}\left(c^{2} x^{2}+1\right)^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{2 b^{2} f^{2} x\left(c^{2} x^{2}+1\right)^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{b^{2} f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \operatorname{arcsinh}(c x)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{b f^{2}\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& -\frac{4 \mathrm{I} b f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{b c f^{2} x^{2}\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& -\frac{2 \mathrm{I} b f^{2} x\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{f^{2} x\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{c^{2} f^{2} x^{3}\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& +\frac{2 f^{2} x\left(c^{2} x^{2}+1\right)^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{2 \mathrm{I} f^{2}\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& -\frac{2 b f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{2 b^{2} f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& +\frac{2 b^{2} f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{b^{2} f^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}
\end{aligned}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{5 / 2} \sqrt{f-\mathrm{I} c f x}} \mathrm{~d} x
$$

Problem 158: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 212 leaves, 7 steps):

$$
\begin{aligned}
& \frac{x\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}+\frac{\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}-\frac{2 b\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x)) \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}} \\
& \quad-\frac{b^{2}\left(c^{2} x^{2}+1\right)^{3 / 2} \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{c(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}}
\end{aligned}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{3 / 2}} \mathrm{~d} x
$$

Problem 159: Unable to integrate problem.

$$
\int \frac{(d+\mathrm{I} c d x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{(f-\mathrm{I} c f x)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 492 leaves, 21 steps):

$$
\begin{aligned}
& \frac{8 d^{4}\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{d^{4}\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{3}}{3 b c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& +\frac{32 b d^{4}\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \ln \left(1+\frac{\mathrm{I}}{c x+\sqrt{c^{2} x^{2}+1}}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{32 b^{2} d^{4}\left(c^{2} x^{2}+1\right)^{5 / 2} \operatorname{polylog}\left(2, \frac{-\mathrm{I}}{c x+\sqrt{c^{2} x^{2}+1}}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& \left.+\frac{4 b d^{4}\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \sec \left(\frac{\pi}{4}+\frac{\mathrm{I} \operatorname{arcsinh}(c x)}{2}\right)^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{8 \mathrm{I} b^{2} d^{4}\left(c^{2} x^{2}+1\right)^{5 / 2} \tan \left(\frac{\pi}{4}+\frac{\mathrm{I} \operatorname{arcsinh}(c x)}{2}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}\right) \\
& +\frac{8 \mathrm{I} d^{4}\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2} \tan \left(\frac{\pi}{4}+\frac{\mathrm{I} \operatorname{arcsinh}(c x)}{2}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& +
\end{aligned}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(d+\mathrm{I} c d x)^{3 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{(f-\mathrm{I} c f x)^{5 / 2}} \mathrm{~d} x
$$

Problem 160: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 662 leaves, 21 steps):

$$
\begin{aligned}
& \frac{\mathrm{I} b^{2} d\left(c^{2} x^{2}+1\right)^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{b^{2} d x\left(c^{2} x^{2}+1\right)^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{b d\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& +\frac{\mathrm{I} b d x\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{\mathrm{I} d\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{d x\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& +\frac{2 d x\left(c^{2} x^{2}+1\right)^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{2 d\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& +\frac{2 \mathrm{I} b d\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \arctan \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{4 b d\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& +\frac{b^{2} d\left(c^{2} x^{2}+1\right)^{5 / 2} \operatorname{polylog}\left(2,-\mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{b^{2} d\left(c^{2} x^{2}+1\right)^{5 / 2} \operatorname{polylog}\left(2, \mathrm{I}\left(c x+\sqrt{c^{2} x^{2}+1}\right)\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& -\frac{2 b^{2} d\left(c^{2} x^{2}+1\right)^{5 / 2} \operatorname{polylog}\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}
\end{aligned}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{3 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \mathrm{~d} x
$$

Problem 161: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 340 leaves, 10 steps):

$$
\begin{aligned}
& -\frac{b^{2} x\left(c^{2} x^{2}+1\right)^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{b\left(c^{2} x^{2}+1\right)^{3 / 2}(a+b \operatorname{arcsinh}(c x))}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{x\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x))^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}+\frac{2 x\left(c^{2} x^{2}+1\right)^{2}(a+b \operatorname{arcsinh}(c x))^{2}}{3(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \\
& \quad+\frac{2\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x))^{2}}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}-\frac{4 b\left(c^{2} x^{2}+1\right)^{5 / 2}(a+b \operatorname{arcsinh}(c x)) \ln \left(1+\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}
\end{aligned}
$$

$-\frac{2 b^{2}\left(c^{2} x^{2}+1\right)^{5 / 2} \text { polylog }\left(2,-\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}\right)}{3 c(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}}$
Result(type 8, 33 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(d+\mathrm{I} c d x)^{5 / 2}(f-\mathrm{I} c f x)^{5 / 2}} \mathrm{~d} x
$$

Problem 164: Result more than twice size of optimal antiderivative.

$$
\int\left(e x^{2}+d\right)^{2}(a+b \operatorname{arcsinh}(c x))^{2} \mathrm{~d} x
$$

Optimal(type 3, 293 leaves, 17 steps):

$$
\begin{aligned}
& 2 b^{2} d^{2} x-\frac{8 b^{2} d e x}{9 c^{2}}+\frac{16 b^{2} e^{2} x}{75 c^{4}}+\frac{4 b^{2} d e x^{3}}{27}-\frac{8 b^{2} e^{2} x^{3}}{225 c^{2}}+\frac{2 b^{2} e^{2} x^{5}}{125}+d^{2} x(a+b \operatorname{arcsinh}(c x))^{2}+\frac{2 d e x^{3}(a+b \operatorname{arcsinh}(c x))^{2}}{3} \\
& \quad+\frac{e^{2} x^{5}(a+b \operatorname{arcsinh}(c x))^{2}}{5}-\frac{2 b d^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{c}+\frac{8 b d e(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{9 c^{3}} \\
& \quad-\frac{16 b e^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{75 c^{5}}-\frac{4 b d e x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{9 c}+\frac{8 b e^{2} x^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{75 c^{3}} \\
& \quad-\frac{2 b e^{2} x^{4}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{25 c} \\
& \text { Result (type 3, 619 leaves) : } \\
& \frac{1}{c}\left(\frac{a^{2}\left(\frac{1}{5} e^{2} c^{5} x^{5}+\frac{2}{3} c^{5} d e x^{3}+x c^{5} d^{2}\right)}{c^{4}}+\frac{1}{c^{4}}\left(b ^ { 2 } \left(\frac { 1 } { 3 3 7 5 } \left(e ^ { 2 } \left(675 \operatorname{arcsinh}(c x)^{2} c^{5} x^{5}-270 \operatorname{arcsinh}(c x) \sqrt{c^{2} x^{2}+1} c^{4} x^{4}+2250 \operatorname{arcsinh}(c x)^{2} c^{3} x^{3}\right.\right.\right.\right.\right.
\end{aligned}
$$

$$
\left.\left.+54 c^{5} x^{5}-1140 \operatorname{arcsinh}(c x) c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+3375 c x \operatorname{arcsinh}(c x)^{2}+380 c^{3} x^{3}-4470 \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)+4470 c x\right)\right)
$$

$$
+\frac{2 c^{2} d e\left(9 \operatorname{arcsinh}(c x)^{2} c^{3} x^{3}-6 \operatorname{arcsinh}(c x) c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+27 c x \operatorname{arcsinh}(c x)^{2}+2 c^{3} x^{3}-42 \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)+42 c x\right)}{27}
$$

$$
-\frac{2 e^{2}\left(9 \operatorname{arcsinh}(c x)^{2} c^{3} x^{3}-6 \operatorname{arcsinh}(c x) c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+27 c x \operatorname{arcsinh}(c x)^{2}+2 c^{3} x^{3}-42 \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)+42 c x\right)}{27}
$$

$$
+d^{2} c^{4}\left(c x \operatorname{arcsinh}(c x)^{2}-2 \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)+2 c x\right)-2 c^{2} d e\left(c x \operatorname{arcsinh}(c x)^{2}-2 \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)+2 c x\right)+e^{2}\left(c x \operatorname{arcsinh}(c x)^{2}\right.
$$

$$
\begin{aligned}
& \left.\left.\left.-2 \sqrt{c^{2} x^{2}+1} \operatorname{arcsinh}(c x)+2 c x\right)\right)\right)+\frac{1}{c^{4}}\left(2 a b \left(\frac{\operatorname{arcsinh}(c x) e^{2} c^{5} x^{5}}{5}+\frac{2 \operatorname{arcsinh}(c x) c^{5} d e x^{3}}{3}+\operatorname{arcsinh}(c x) d^{2} c^{5} x\right.\right. \\
& \left.\left.\left.-\frac{e^{2}\left(\frac{c^{4} x^{4} \sqrt{c^{2} x^{2}+1}}{5}-\frac{4 c^{2} x^{2} \sqrt{c^{2} x^{2}+1}}{15}+\frac{8 \sqrt{c^{2} x^{2}+1}}{15}\right)}{5}-d^{2} c^{4} \sqrt{c^{2} x^{2}+1}-\frac{2 c^{2} d e\left(\frac{c^{2} x^{2} \sqrt{c^{2} x^{2}+1}}{3}-\frac{2 \sqrt{c^{2} x^{2}+1}}{3}\right)}{3}\right)\right)\right)
\end{aligned}
$$

Problem 165: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{e x^{2}+d} \mathrm{~d} x
$$

Optimal(type 4, 719 leaves, 22 steps):

$$
\begin{aligned}
& \frac{(a+b \operatorname{arcsinh}(c x))^{2} \ln \left(1-\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}-\sqrt{-c^{2} d+e}}\right)}{2 \sqrt{-d} \sqrt{e}}-\frac{(a+b \operatorname{arcsinh}(c x))^{2} \ln \left(1+\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}-\sqrt{-c^{2} d+e}}\right)}{2 \sqrt{-d} \sqrt{e}} \\
& +\frac{(a+b \operatorname{arcsinh}(c x))^{2} \ln \left(1-\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}+\sqrt{-c^{2} d+e}}\right)}{2 \sqrt{-d} \sqrt{e}}-\frac{(a+b \operatorname{arcsinh}(c x))^{2} \ln \left(1+\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}+\sqrt{-c^{2} d+e}}\right)}{2 \sqrt{-d} \sqrt{e}} \\
& -\frac{b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}-\sqrt{-c^{2} d+e}}\right)}{\sqrt{-d} \sqrt{e}}+\frac{b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}-\sqrt{-c^{2} d+e}}\right)}{\sqrt{-d} \sqrt{e}} \\
& -\frac{b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}+\sqrt{-c^{2} d+e}}\right)}{\sqrt{e}}+\frac{b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2, \frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{\left.c \sqrt{-d}+\sqrt{-c^{2} d+e}\right)}\right.}{\sqrt{e}} \\
& +\frac{b^{2} \text { polylog }\left(3,-\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}-\sqrt{-c^{2} d+e}}\right)}{\sqrt{-d} \sqrt{e}}-\frac{b^{2} \operatorname{polylog}\left(3, \frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}-\sqrt{-c^{2} d+e}}\right)}{\sqrt{-d} \sqrt{e}}+\frac{b^{2} \operatorname{polylog}\left(3,-\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}+\sqrt{-c^{2} d+e}}\right)}{\sqrt{-d} \sqrt{e}}
\end{aligned}
$$

$$
-\frac{b^{2} \text { polylog }\left(3, \frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) \sqrt{e}}{c \sqrt{-d}+\sqrt{-c^{2} d+e}}\right)}{\sqrt{-d} \sqrt{e}}
$$

Result(type 8, 22 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{e x^{2}+d} \mathrm{~d} x
$$

Problem 170: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(e x^{2}+d\right)^{2}}{(a+b \operatorname{arcsinh}(c x))^{2}} \mathrm{~d} x
$$

Optimal(type 4, 469 leaves, 26 steps):

$$
\begin{aligned}
& \frac{d^{2} \cosh \left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right)}{b^{2} c}-\frac{d e \cosh \left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right)}{2 b^{2} c^{3}}+\frac{e^{2} \cosh \left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right)}{8 b^{2} c^{5}} \\
& +\frac{3 d e \cosh \left(\frac{3 a}{b}\right) \operatorname{Shi}\left(\frac{3(a+b \operatorname{arcsinh}(c x))}{b}\right)}{2 b^{2} c^{3}}-\frac{9 e^{2} \cosh \left(\frac{3 a}{b}\right) \operatorname{Shi}\left(\frac{3(a+b \operatorname{arcsinh}(c x))}{b}\right)}{16 b^{2} c^{5}}+\frac{5 e^{2} \cosh \left(\frac{5 a}{b}\right) \operatorname{Shi}\left(\frac{5(a+b \operatorname{arcsinh}(c x))}{b}\right)}{16 b^{2} c^{5}} \\
& -\frac{d^{2} \operatorname{Chi}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right) \sinh \left(\frac{a}{b}\right)}{b^{2} c}+\frac{d e \operatorname{Chi}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right) \sinh \left(\frac{a}{b}\right)}{2 b^{2} c^{3}}-\frac{e^{2} \operatorname{Chi}\left(\frac{a+b \operatorname{arcsinh}(c x)}{b}\right) \sinh \left(\frac{a}{b}\right)}{b} \\
& -\frac{3 d e \operatorname{Chi}\left(\frac{3(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{3 a}{b}\right)}{2 b^{2} c^{3}}+\frac{9 e^{2} \operatorname{Chi}\left(\frac{3(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{3 a}{b}\right)}{16 b^{2} c^{5}}-\frac{5 e^{2} \operatorname{Chi}\left(\frac{5(a+b \operatorname{arcsinh}(c x))}{b}\right) \sinh \left(\frac{5 a}{b}\right)}{16 b^{2} c^{5}}
\end{aligned}
$$

$$
-\frac{d^{2} \sqrt{c^{2} x^{2}+1}}{b c(a+b \operatorname{arcsinh}(c x))}-\frac{2 d e x^{2} \sqrt{c^{2} x^{2}+1}}{b c(a+b \operatorname{arcsinh}(c x))}-\frac{e^{2} x^{4} \sqrt{c^{2} x^{2}+1}}{b c(a+b \operatorname{arcsinh}(c x))}
$$

Result(type 4, 1035 leaves):

$$
\begin{aligned}
& \frac{1}{c}\left(\frac{\left(16 c^{5} x^{5}-16 c^{4} x^{4} \sqrt{c^{2} x^{2}+1}+20 c^{3} x^{3}-12 c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+5 c x-\sqrt{c^{2} x^{2}+1}\right) e^{2}}{32 c^{4} b(a+b \operatorname{arcsinh}(c x))}+\frac{5 e^{2} \mathrm{e}^{\frac{5 a}{b}} \operatorname{Ei}_{1}\left(5 \operatorname{arcsinh}(c x)+\frac{5 a}{b}\right)}{32 c^{4} b^{2}}\right. \\
& \quad-\frac{e^{2}\left(16 c^{5} x^{5}+20 c^{3} x^{3}+16 c^{4} x^{4} \sqrt{c^{2} x^{2}+1}+5 c x+12 c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+\sqrt{c^{2} x^{2}+1}\right)}{32 c^{4} b(a+b \operatorname{arcsinh}(c x))}-\frac{5 e^{2} \mathrm{e}^{-\frac{5 a}{b}} \operatorname{Ei}_{1}\left(-5 \operatorname{arcsinh}(c x)-\frac{5 a}{b}\right)}{32 c^{4} b^{2}}
\end{aligned}
$$

$$
+\frac{\left(c x-\sqrt{c^{2} x^{2}+1}\right) d^{2}}{2 b(a+b \operatorname{arcsinh}(c x))}+\frac{d^{2} \mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_{1}\left(\operatorname{arcsinh}(c x)+\frac{a}{b}\right)}{2 b^{2}}-\frac{\left(c x-\sqrt{c^{2} x^{2}+1}\right) d e}{4 c^{2} b(a+b \operatorname{arcsinh}(c x))}-\frac{d e \mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_{1}\left(\operatorname{arcsinh}(c x)+\frac{a}{b}\right)}{4 c^{2} b^{2}}
$$

$$
\begin{aligned}
& +\frac{\left(c x-\sqrt{c^{2} x^{2}+1}\right) e^{2}}{16 c^{4} b(a+b \operatorname{arcsinh}(c x))}+\frac{e^{2} \mathrm{e}^{\frac{a}{b}} \operatorname{Ei}_{1}\left(\operatorname{arcsinh}(c x)+\frac{a}{b}\right)}{16 c^{4} b^{2}}-\frac{d^{2}\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{2 b(a+b \operatorname{arcsinh}(c x))}-\frac{d^{2} \mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(c x)-\frac{a}{b}\right)}{2 b^{2}} \\
& +\frac{d e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{4 c^{2} b(a+b \operatorname{arcsinh}(c x))}+\frac{d e \mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(c x)-\frac{a}{b}\right)}{4 c^{2} b^{2}}-\frac{e^{2}\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{16 c^{4} b(a+b \operatorname{arcsinh}(c x))}-\frac{e^{2} \mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsinh}(c x)-\frac{a}{b}\right)}{16 c^{4} b^{2}} \\
& +\frac{\left(4 c^{3} x^{3}-4 c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+3 c x-\sqrt{c^{2} x^{2}+1}\right) d e}{4 c^{2} b(a+b \operatorname{arcsinh}(c x))}-\frac{3\left(4 c^{3} x^{3}-4 c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+3 c x-\sqrt{c^{2} x^{2}+1}\right) e^{2}}{32 c^{4} b(a+b \operatorname{arcsinh}(c x))} \\
& +\frac{3 e \mathrm{e}^{\frac{3 a}{b}} \operatorname{Ei}_{1}\left(3 \operatorname{arcsinh}(c x)+\frac{3 a}{b}\right) d}{4 c^{2} b^{2}}-\frac{9 e^{2} \mathrm{e}^{\frac{3 a}{b}} \operatorname{Ei}_{1}\left(3 \operatorname{arcsinh}(c x)+\frac{3 a}{b}\right)}{32 c^{4} b^{2}}-\frac{e\left(4 c^{3} x^{3}+3 c x+4 c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+\sqrt{c^{2} x^{2}+1}\right) d}{4 c^{2} b(a+b \operatorname{arcsinh}(c x))} \\
& +\frac{3 e^{2}\left(4 c^{3} x^{3}+3 c x+4 c^{2} x^{2} \sqrt{c^{2} x^{2}+1}+\sqrt{c^{2} x^{2}+1}\right)}{32 c^{4} b(a+b \operatorname{arcsinh}(c x))}-\frac{3 e \mathrm{e}}{\mathrm{Ei}_{1}\left(-3 \operatorname{arcsinh}(c x)-\frac{3 a}{b}\right) d}+9 e^{2} \mathrm{e}^{-\frac{3 a}{b}} \operatorname{\operatorname {Ei}_{1}(-3\operatorname {arcsinh}(cx)-\frac {3a}{b})}
\end{aligned}
$$

Problem 172: Unable to integrate problem.

$$
\int \frac{e x^{2}+d}{\sqrt{a+b \operatorname{arcsinh}(c x)}} \mathrm{d} x
$$

Optimal(type 4, 219 leaves, 21 steps):
$\frac{e \mathrm{e}^{\frac{3 a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{24 c^{3} \sqrt{b}}+\frac{e \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{24 c^{3} \mathrm{e}^{\frac{3 a}{b}} \sqrt{b}}+\frac{d \mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{2 c \sqrt{b}}$
$-\frac{e \mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{8 c^{3} \sqrt{b}}+\frac{d \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{2 c \mathrm{e}^{\frac{a}{b}} \sqrt{b}}-\frac{e \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{8 c^{3} \mathrm{e}^{\frac{a}{b}} \sqrt{b}}$

Result(type 8, 20 leaves):

$$
\int \frac{e x^{2}+d}{\sqrt{a+b \operatorname{arcsinh}(c x)}} \mathrm{d} x
$$

[^1]$$
\int \frac{e x^{2}+d}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 281 leaves, 21 steps):


$$
-\frac{e \mathrm{e}^{\frac{3 a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{4 b^{3 / 2} c^{3}}+\frac{e \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(c x)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{4 b^{3 / 2} c^{3} \mathrm{e}^{\frac{3 a}{b}}}-\frac{2 d \sqrt{c^{2} x^{2}+1}}{b c \sqrt{a+b \operatorname{arcsinh}(c x)}}-\frac{2 e x^{2} \sqrt{c^{2} x^{2}+1}}{b c \sqrt{a+b \operatorname{arcsinh}(c x)}}
$$

Result(type 8, 20 leaves):

$$
\int \frac{e x^{2}+d}{(a+b \operatorname{arcsinh}(c x))^{3 / 2}} \mathrm{~d} x
$$

Problem 174: Unable to integrate problem.

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{\left(e x^{2}+d\right)^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 193 leaves, 8 steps):

$$
\begin{aligned}
& \frac{x(a+b \operatorname{arcsinh}(c x))}{5 d\left(e x^{2}+d\right)^{5 / 2}}+\frac{4 x(a+b \operatorname{arcsinh}(c x))}{15 d^{2}\left(e x^{2}+d\right)^{3 / 2}}-\frac{8 b \operatorname{arctanh}\left(\frac{\sqrt{e} \sqrt{c^{2} x^{2}+1}}{c \sqrt{e x^{2}+d}}\right)}{15 d^{3} \sqrt{e}}-\frac{b c \sqrt{c^{2} x^{2}+1}}{15 d\left(c^{2} d-e\right)\left(e x^{2}+d\right)^{3 / 2}}+\frac{8 x(a+b \operatorname{arcsinh}(c x))}{15 d^{3} \sqrt{e x^{2}+d}} \\
& \quad-\frac{2 b c\left(3 c^{2} d-2 e\right) \sqrt{c^{2} x^{2}+1}}{15 d^{2}\left(c^{2} d-e\right)^{2} \sqrt{e x^{2}+d}}
\end{aligned}
$$

Result(type 8, 20 leaves):

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{\left(e x^{2}+d\right)^{7 / 2}} \mathrm{~d} x
$$

Test results for the 100 problems in "7.1.5 Inverse hyperbolic sine functions.txt"
Problem 2: Unable to integrate problem.

$$
\int \frac{\operatorname{arcsinh}(c x)^{3}}{e x+d} \mathrm{~d} x
$$

Optimal(type 4, 402 leaves, 12 steps):

$$
\begin{aligned}
& -\frac{\operatorname{arcsinh}(c x)^{4}}{4 e}+\frac{\operatorname{arcsinh}(c x)^{3} \ln \left(1+\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}+\frac{\operatorname{arcsinh}(c x)^{3} \ln \left(1+\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}+\frac{3 \operatorname{arcsinh}(c x)^{2} \operatorname{polylog}\left(2,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}+\frac{3 \operatorname{arcsinh}(c x)^{2} \operatorname{polylog}\left(2,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}+\frac{e \operatorname{ercsinh}(c x) \operatorname{polylog}\left(3,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}-\frac{6 \operatorname{arcsinh}(c x) \operatorname{polylog}\left(3,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}+\frac{6 \operatorname{polylog}\left(4,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e} \\
& \\
& - \\
& \quad+\frac{6 \operatorname{polylog}\left(4,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}
\end{aligned}
$$

Result(type 8, 16 leaves):

$$
\int \frac{\operatorname{arcsinh}(c x)^{3}}{e x+d} \mathrm{~d} x
$$

Problem 4: Result more than twice size of optimal antiderivative.

$$
\int \frac{a+b \operatorname{arcsinh}(c x)}{(e x+d)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 117 leaves, 4 steps):

$$
\frac{-a-b \operatorname{arcsinh}(c x)}{2 e(e x+d)^{2}}-\frac{b c^{3} d \operatorname{arctanh}\left(\frac{-c^{2} d x+e}{\sqrt{d^{2} c^{2}+e^{2}} \sqrt{c^{2} x^{2}+1}}\right)}{2 e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}}-\frac{b c \sqrt{c^{2} x^{2}+1}}{2\left(d^{2} c^{2}+e^{2}\right)(e x+d)}
$$

Result(type 3, 278 leaves):

$$
-\frac{c^{2} a}{2(c e x+c d)^{2} e}-\frac{c^{2} b \operatorname{arcsinh}(c x)}{2(c e x+c d)^{2} e}-\frac{c^{2} b \sqrt{\left(c x+\frac{c d}{e}\right)^{2}-\frac{2 c d\left(c x+\frac{c d}{e}\right)}{e}+\frac{d^{2} c^{2}+e^{2}}{e^{2}}}}{2 e\left(d^{2} c^{2}+e^{2}\right)\left(c x+\frac{c d}{e}\right)}
$$

$$
-\frac{c^{3} b d \ln \left(\frac{\frac{2\left(d^{2} c^{2}+e^{2}\right)}{e^{2}}-\frac{2 c d\left(c x+\frac{c d}{e}\right)}{e}+2 \sqrt{\frac{d^{2} c^{2}+e^{2}}{e^{2}}} \sqrt{\left(c x+\frac{c d}{e}\right)^{2}-\frac{2 c d\left(c x+\frac{c d}{e}\right)}{e}+\frac{d^{2} c^{2}+e^{2}}{e^{2}}}}{c x+\frac{c d}{e}}\right)}{2 e^{2}\left(d^{2} c^{2}+e^{2}\right) \sqrt{\frac{d^{2} c^{2}+e^{2}}{e^{2}}}}
$$

Problem 6: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{e x+d} \mathrm{~d} x
$$

Optimal(type 4, 331 leaves, 10 steps):

$$
\begin{aligned}
& -\frac{(a+b \operatorname{arcsinh}(c x))^{3}}{3 b e}+\frac{(a+b \operatorname{arcsinh}(c x))^{2} \ln \left(1+\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}+\frac{(a+b \operatorname{arcsinh}(c x))^{2} \ln \left(1+\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{\left.c d+\sqrt{d^{2} c^{2}+e^{2}}\right)}\right.}{e}+\frac{2 b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}+\frac{2 b(a+b \operatorname{arcsinh}(c x)) \operatorname{polylog}\left(2,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e} \\
& +\frac{2 b^{2} \operatorname{polylog}\left(3,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}-\frac{2 b^{2} \operatorname{polylog}\left(3,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e}
\end{aligned}
$$

Result(type 8, 20 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{e x+d} \mathrm{~d} x
$$

Problem 8: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{(e x+d)^{3}} \mathrm{~d} x
$$

Optimal(type 4, 365 leaves, 13 steps):

$$
-\frac{(a+b \operatorname{arcsinh}(c x))^{2}}{2 e(e x+d)^{2}}+\frac{b^{2} c^{2} \ln (e x+d)}{e\left(d^{2} c^{2}+e^{2}\right)}+\frac{b c^{3} d(a+b \operatorname{arcsinh}(c x)) \ln \left(1+\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}}
$$

$$
\begin{aligned}
& \left.\frac{b c^{3} d(a+b \operatorname{arcsinh}(c x)) \ln \left(1+\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}}+\frac{b^{2} c^{3} d \operatorname{polylog}\left(2,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d-\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}}\right) \\
- & \frac{b^{2} c^{3} d \operatorname{polylog}\left(2,-\frac{e\left(c x+\sqrt{c^{2} x^{2}+1}\right)}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}}-\frac{b c(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} x^{2}+1}}{\left(d^{2} c^{2}+e^{2}\right)(e x+d)}
\end{aligned}
$$

Result(type 4, 1012 leaves):

$$
\begin{aligned}
& -\frac{c^{2} a^{2}}{2(c e x+c d)^{2} e}-\frac{c^{4} b^{2} \operatorname{arcsinh}(c x)^{2} d^{2}}{2 e(c e x+c d)^{2}\left(d^{2} c^{2}+e^{2}\right)}-\frac{c^{3} b^{2} \operatorname{arcsinh}(c x) e \sqrt{c^{2} x^{2}+1} x}{(c e x+c d)^{2}\left(d^{2} c^{2}+e^{2}\right)}-\frac{c^{3} b^{2} \operatorname{arcsinh}(c x) d \sqrt{c^{2} x^{2}+1}}{(c e x+c d)^{2}\left(d^{2} c^{2}+e^{2}\right)}+\frac{c^{4} b^{2} \operatorname{arcsinh}(c x) e x^{2}}{(c e x+c d)^{2}\left(d^{2} c^{2}+e^{2}\right)} \\
& +\frac{2 c^{4} b^{2} \operatorname{arcsinh}(c x) x d}{(c e x+c d)^{2}\left(d^{2} c^{2}+e^{2}\right)}+\frac{c^{4} b^{2} \operatorname{arcsinh}(c x) d^{2}}{e(c e x+c d)^{2}\left(d^{2} c^{2}+e^{2}\right)}-\frac{c^{2} b^{2} \operatorname{arcsinh}(c x)^{2} e}{2(c e x+c d)^{2}\left(d^{2} c^{2}+e^{2}\right)}-\frac{2 c^{2} b^{2} \ln \left(c x+\sqrt{c^{2} x^{2}+1}\right)}{e\left(d^{2} c^{2}+e^{2}\right)} \\
& +\frac{c^{2} b^{2} \ln \left(2 c d\left(c x+\sqrt{c^{2} x^{2}+1}\right)+e\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}-e\right)}{e\left(d^{2} c^{2}+e^{2}\right)}+\frac{c^{3} b^{2} d \operatorname{arcsinh}(c x) \ln \left(\frac{-\left(c x+\sqrt{c^{2} x^{2}+1}\right) e-c d+\sqrt{d^{2} c^{2}+e^{2}}}{-c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}} \\
& -\frac{c^{3} b^{2} d \operatorname{arcsinh}(c x) \ln \left(\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) e+c d+\sqrt{d^{2} c^{2}+e^{2}}}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}}+\frac{c^{3} b^{2} d \operatorname{dilog}\left(\frac{-\left(c x+\sqrt{c^{2} x^{2}+1}\right) e-c d+\sqrt{d^{2} c^{2}+e^{2}}}{-c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}} \\
& -\frac{c^{3} b^{2} d \operatorname{dilog}\left(\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) e+c d+\sqrt{d^{2} c^{2}+e^{2}}}{c d+\sqrt{d^{2} c^{2}+e^{2}}}\right)}{e\left(d^{2} c^{2}+e^{2}\right)^{3 / 2}}-\frac{c^{2} a b \operatorname{arcsinh}(c x)}{(c e x+c d)^{2} e}-\frac{c^{2} a b \sqrt{\left(c x+\frac{c d}{e}\right)^{2}-\frac{2 c d\left(c x+\frac{c d}{e}\right)}{e}+\frac{d^{2} c^{2}+e^{2}}{e^{2}}}}{e\left(d^{2} c^{2}+e^{2}\right)\left(c x+\frac{c d}{e}\right)} \\
& c^{3} a b d \ln \left(\frac{\frac{2\left(d^{2} c^{2}+e^{2}\right)}{e^{2}}-\frac{2 c d\left(c x+\frac{c d}{e}\right)}{e}+2 \sqrt{\frac{d^{2} c^{2}+e^{2}}{e^{2}}} \sqrt{\left(c x+\frac{c d}{e}\right)^{2}-\frac{2 c d\left(c x+\frac{c d}{e}\right)}{e}+\frac{d^{2} c^{2}+e^{2}}{e^{2}}}}{c x+\frac{c d}{e}}\right) \\
& e^{2}\left(d^{2} c^{2}+e^{2}\right) \sqrt{\frac{d^{2} c^{2}+e^{2}}{e^{2}}}
\end{aligned}
$$

Problem 12: Result more than twice size of optimal antiderivative.

$$
\int(g x+f)^{2}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d} \mathrm{~d} x
$$

Optimal(type 3, 373 leaves, 13 steps):
$\frac{f^{2} x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{2}+\frac{g^{2} x(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{8 c^{2}}+\frac{g^{2} x^{3}(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{4}$

$$
+\frac{2 f g\left(c^{2} x^{2}+1\right)(a+b \operatorname{arcsinh}(c x)) \sqrt{c^{2} d x^{2}+d}}{3 c^{2}}-\frac{2 b f g x \sqrt{c^{2} d x^{2}+d}}{3 c \sqrt{c^{2} x^{2}+1}}-\frac{b c f^{2} x^{2} \sqrt{c^{2} d x^{2}+d}}{4 \sqrt{c^{2} x^{2}+1}}-\frac{b g^{2} x^{2} \sqrt{c^{2} d x^{2}+d}}{16 c \sqrt{c^{2} x^{2}+1}}-\frac{2 b c f g x^{3} \sqrt{c^{2} d x^{2}+d}}{9 \sqrt{c^{2} x^{2}+1}}
$$

$$
-\frac{b c g^{2} x^{4} \sqrt{c^{2} d x^{2}+d}}{16 \sqrt{c^{2} x^{2}+1}}+\frac{f^{2}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{4 b c \sqrt{c^{2} x^{2}+1}}-\frac{g^{2}(a+b \operatorname{arcsinh}(c x))^{2} \sqrt{c^{2} d x^{2}+d}}{16 b c^{3} \sqrt{c^{2} x^{2}+1}}
$$

Result(type 3, 790 leaves):
$\frac{a f^{2} x \sqrt{c^{2} d x^{2}+d}}{2}+\frac{a f^{2} d \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{2 \sqrt{c^{2} d}}+\frac{a g^{2} x\left(c^{2} d x^{2}+d\right)^{3 / 2}}{4 c^{2} d}-\frac{a g^{2} x \sqrt{c^{2} d x^{2}+d}}{8 c^{2}}-\frac{a g^{2} d \ln \left(\frac{x c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d x^{2}+d}\right)}{8 c^{2} \sqrt{c^{2} d}}$

$$
\begin{aligned}
& +\frac{2 a f g\left(c^{2} d x^{2}+d\right)^{3 / 2}}{3 c^{2} d}-\frac{2 b \sqrt{d\left(c^{2} x^{2}+1\right)} f g c x^{3}}{9 \sqrt{c^{2} x^{2}+1}}+\frac{4 b \sqrt{d\left(c^{2} x^{2}+1\right)} f g \operatorname{arcsinh}(c x) x^{2}}{3\left(c^{2} x^{2}+1\right)}-\frac{2 b \sqrt{d\left(c^{2} x^{2}+1\right)} f g x}{3 c \sqrt{c^{2} x^{2}+1}} \\
& +\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} f^{2}}{4 \sqrt{c^{2} x^{2}+1} c}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} \operatorname{arcsinh}(c x)^{2} g^{2}}{16 \sqrt{c^{2} x^{2}+1} c^{3}}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} g^{2}}{128 c^{3} \sqrt{c^{2} x^{2}+1}}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} f^{2}}{8 c \sqrt{c^{2} x^{2}+1}}
\end{aligned}
$$

$$
+\frac{2 b \sqrt{d\left(c^{2} x^{2}+1\right)} f g c^{2} \operatorname{arcsinh}(c x) x^{4}}{3\left(c^{2} x^{2}+1\right)}+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} f^{2} c^{2} \operatorname{arcsinh}(c x) x^{3}}{2\left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} f^{2} c x^{2}}{4 \sqrt{c^{2} x^{2}+1}}+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} f^{2} \operatorname{arcsinh}(c x) x}{2\left(c^{2} x^{2}+1\right)}
$$

$$
+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} g^{2} c^{2} \operatorname{arcsinh}(c x) x^{5}}{4\left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} g^{2} c x^{4}}{16 \sqrt{c^{2} x^{2}+1}}+\frac{3 b \sqrt{d\left(c^{2} x^{2}+1\right)} g^{2} \operatorname{arcsinh}(c x) x^{3}}{8\left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} g^{2} x^{2}}{16 c \sqrt{c^{2} x^{2}+1}}
$$

$$
+\frac{b \sqrt{d\left(c^{2} x^{2}+1\right)} g^{2} \operatorname{arcsinh}(c x) x}{8 c^{2}\left(c^{2} x^{2}+1\right)}+\frac{2 b \sqrt{d\left(c^{2} x^{2}+1\right)} f g \operatorname{arcsinh}(c x)}{3 c^{2}\left(c^{2} x^{2}+1\right)}
$$

Problem 18: Unable to integrate problem.

$$
\int \frac{\ln \left(h(g x+f)^{m}\right)}{\sqrt{c^{2} x^{2}+1}} d x
$$

Optimal(type 4, 223 leaves, 9 steps):
$\frac{m \operatorname{arcsinh}(c x)^{2}}{2 c}+\frac{\operatorname{arcsinh}(c x) \ln \left(h(g x+f)^{m}\right)}{c}-\frac{m \operatorname{arcsinh}(c x) \ln \left(1+\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) g}{c f-\sqrt{c^{2} f^{2}+g^{2}}}\right)}{c}-\frac{m \operatorname{arcsinh}(c x) \ln \left(1+\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) g}{c f+\sqrt{c^{2} f^{2}+g^{2}}}\right)}{c}$
$-\frac{m \text { polylog }\left(2,-\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) g}{c f-\sqrt{c^{2} f^{2}+g^{2}}}\right)}{c}-\frac{m \text { polylog }\left(2,-\frac{\left(c x+\sqrt{c^{2} x^{2}+1}\right) g}{c f+\sqrt{c^{2} f^{2}+g^{2}}}\right)}{c}$
Result(type 8, 24 leaves):

$$
\int \frac{\ln \left(h(g x+f)^{m}\right)}{\sqrt{c^{2} x^{2}+1}} \mathrm{~d} x
$$

Problem 24: Unable to integrate problem.

$$
\int \frac{\operatorname{arcsinh}(b x+a)^{3}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 4, 550 leaves, 21 steps):

$$
\begin{aligned}
& \left.-\frac{3 b^{2} \operatorname{arcsinh}(b x+a)^{2}}{2\left(a^{2}+1\right)}-\frac{\operatorname{arcsinh}(b x+a)^{3}}{2 x^{2}}+\frac{3 b^{2} \operatorname{arcsinh}(b x+a) \ln \left(1-\frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a-\sqrt{a^{2}+1}}\right)}{a^{2}+1}\right) \\
& \\
& \quad+\frac{3 a b^{2} \operatorname{arcsinh}(b x+a)^{2} \ln \left(1-\frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a-\sqrt{a^{2}+1}}\right)}{2\left(a^{2}+1\right)^{3 / 2}}+\frac{3 b^{2} \operatorname{arcsinh}(b x+a) \ln \left(1-\frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a+\sqrt{a^{2}+1}}\right)}{a^{2}+1}
\end{aligned}
$$

$$
-\frac{3 a b^{2} \operatorname{arcsinh}(b x+a)^{2} \ln \left(1-\frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a+\sqrt{a^{2}+1}}\right)}{2\left(a^{2}+1\right)^{3 / 2}}+\frac{3 b^{2} \operatorname{polylog}\left(2, \frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a-\sqrt{a^{2}+1}}\right)}{a^{2}+1}
$$

$$
+\frac{3 a b^{2} \operatorname{arcsinh}(b x+a) \operatorname{polylog}\left(2, \frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a-\sqrt{a^{2}+1}}\right)}{\left(a^{2}+1\right)^{3 / 2}}+\frac{3 b^{2} \operatorname{polylog}\left(2, \frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a+\sqrt{a^{2}+1}}\right)}{a^{2}+1}
$$

$$
-\frac{3 a b^{2} \operatorname{arcsinh}(b x+a) \operatorname{polylog}\left(2, \frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a+\sqrt{a^{2}+1}}\right)}{\left(a^{2}+1\right)^{3 / 2}}-\frac{3 a b^{2} \operatorname{polylog}\left(3, \frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a-\sqrt{a^{2}+1}}\right)}{\left(a^{2}+1\right)^{3 / 2}}
$$

$$
+\frac{3 a b^{2} \text { polylog }\left(3, \frac{b x+a+\sqrt{1+(b x+a)^{2}}}{a+\sqrt{a^{2}+1}}\right)}{\left(a^{2}+1\right)^{3 / 2}}-\frac{3 b \operatorname{arcsinh}(b x+a)^{2} \sqrt{1+(b x+a)^{2}}}{2\left(a^{2}+1\right) x}
$$

Result(type 8, 14 leaves):

$$
\int \frac{\operatorname{arcsinh}(b x+a)^{3}}{x^{3}} \mathrm{~d} x
$$

Problem 28: Unable to integrate problem.

$$
\int \sqrt{a+b \operatorname{arcsinh}(d x+c)} \mathrm{d} x
$$

Optimal(type 4, 92 leaves, 8 steps):

$$
\frac{\mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{4 d}-\frac{\operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{4 d \mathrm{e}^{\frac{a}{b}}}+\frac{(d x+c) \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{d}
$$

Result(type 8, 14 leaves):

$$
\int \sqrt{a+b \operatorname{arcsinh}(d x+c)} \mathrm{d} x
$$

Problem 29: Unable to integrate problem.

$$
\int x(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \mathrm{~d} x
$$

Optimal(type 4, 263 leaves, 16 steps):
$-\frac{c(d x+c)(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}{d^{2}}+\frac{(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \cosh (2 \operatorname{arcsinh}(d x+c))}{4 d^{2}}$

$-\frac{3 b \sinh (2 \operatorname{arcsinh}(d x+c)) \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{16 d^{2}}+\frac{3 b c \sqrt{1+(d x+c)^{2}} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{2 d^{2}}$
Result(type 8, 16 leaves):

$$
\int x(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \mathrm{~d} x
$$

Problem 30: Unable to integrate problem.

$$
\int(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \mathrm{~d} x
$$

Optimal(type 4, 121 leaves, 9 steps):


$$
-\frac{3 b \sqrt{1+(d x+c)^{2}} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{2 d}
$$

Result (type 8, 14 leaves):

$$
\int(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \mathrm{~d} x
$$

Problem 31: Unable to integrate problem.

$$
\int \frac{1}{(a+b \operatorname{arcsinh}(d x+c))^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 158 leaves, 10 steps):
$-\frac{4(d x+c)}{15 b^{2} d(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}-\frac{4 \mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{15 b^{7 / 2} d}+\frac{4 \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{15 b^{7 / 2} d \mathrm{e}^{\frac{a}{b}}}$

$$
-\frac{2 \sqrt{1+(d x+c)^{2}}}{5 b d(a+b \operatorname{arcsinh}(d x+c))^{5 / 2}}-\frac{8 \sqrt{1+(d x+c)^{2}}}{15 b^{3} d \sqrt{a+b \operatorname{arcsinh}(d x+c)}}
$$

Result(type 8, 14 leaves):

$$
\int \frac{1}{(a+b \operatorname{arcsinh}(d x+c))^{7 / 2}} \mathrm{~d} x
$$

Problem 38: Unable to integrate problem.

$$
\int(d e x+c e)^{m}(a+b \operatorname{arcsinh}(d x+c))^{2} \mathrm{~d} x
$$

Optimal(type 5, 169 leaves, 3 steps):
$\frac{(e(d x+c))^{1+m}(a+b \operatorname{arcsinh}(d x+c))^{2}}{d e(1+m)}-\frac{2 b(e(d x+c))^{2+m}(a+b \operatorname{arcsinh}(d x+c)) \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-(d x+c)^{2}\right)}{d e^{2}(1+m)(2+m)}$

$$
+\frac{2 b^{2}(e(d x+c))^{3+m} \text { HypergeometricPFQ }\left(\left[1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right],\left[2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right],-(d x+c)^{2}\right)}{d e^{3}(1+m)(2+m)(3+m)}
$$

Result(type 8, 25 leaves):

$$
\int(d e x+c e)^{m}(a+b \operatorname{arcsinh}(d x+c))^{2} \mathrm{~d} x
$$

Problem 41: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(d x+c))^{3}}{d e x+c e} \mathrm{~d} x
$$

Optimal(type 4, 179 leaves, 9 steps):
$\frac{(a+b \operatorname{arcsinh}(d x+c))^{4}}{4 b d e}+\frac{(a+b \operatorname{arcsinh}(d x+c))^{3} \ln \left(1-\frac{1}{\left(d x+c+\sqrt{1+(d x+c)^{2}}\right)^{2}}\right)}{d e}$


$$
-\frac{3 b^{2}(a+b \operatorname{arcsinh}(d x+c)) \text { polylog }\left(3, \frac{1}{\left(d x+c+\sqrt{1+(d x+c)^{2}}\right)^{2}}\right)}{2 d e}-\frac{3 b^{3} \operatorname{polylog}\left(4, \frac{1}{\left(d x+c+\sqrt{1+(d x+c)^{2}}\right)^{2}}\right)}{4 d e}
$$

Result(type 4, 735 leaves):
$\frac{a^{3} \ln (d x+c)}{d e}-\frac{b^{3} \operatorname{arcsinh}(d x+c)^{4}}{4 d e}+\frac{b^{3} \operatorname{arcsinh}(d x+c)^{3} \ln \left(1+d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e}$
$+\frac{3 b^{3} \operatorname{arcsinh}(d x+c)^{2} \text { polylog }\left(2,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e}-\frac{6 b^{3} \operatorname{arcsinh}(d x+c) \operatorname{polylog}\left(3,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e}$
$+\frac{6 b^{3} \operatorname{polylog}\left(4,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e}+\frac{b^{3} \operatorname{arcsinh}(d x+c)^{3} \ln \left(1-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e}$
$+\frac{3 b^{3} \operatorname{arcsinh}(d x+c)^{2} \operatorname{polylog}\left(2, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e}-\frac{6 b^{3} \operatorname{arcsinh}(d x+c) \operatorname{polylog}\left(3, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e}$
$+\frac{6 b^{3} \operatorname{polylog}\left(4, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e}-\frac{a b^{2} \operatorname{arcsinh}(d x+c)^{3}}{d e}+\frac{3 a b^{2} \operatorname{arcsinh}(d x+c)^{2} \ln \left(1+d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e}$
$+\frac{6 a b^{2} \operatorname{arcsinh}(d x+c) \operatorname{polylog}\left(2,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e}-\frac{6 a b^{2} \operatorname{polylog}\left(3,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e}$
$+\frac{3 a b^{2} \operatorname{arcsinh}(d x+c)^{2} \ln \left(1-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e}+\frac{6 a b^{2} \operatorname{arcsinh}(d x+c) \operatorname{polylog}\left(2, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e}$

$$
\begin{aligned}
& -\frac{6 a b^{2} \operatorname{polylog}\left(3, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e}-\frac{3 a^{2} b \operatorname{arcsinh}(d x+c)^{2}}{2 d e}+\frac{3 a^{2} b \operatorname{arcsinh}(d x+c) \ln \left(1+d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e} \\
& +\frac{3 a^{2} b \operatorname{polylog}\left(2,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e}+\frac{3 a^{2} b \operatorname{arcsinh}(d x+c) \ln \left(1-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e} \\
& +\frac{3 a^{2} b \operatorname{polylog}\left(2, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e}
\end{aligned}
$$

Problem 42: Result more than twice size of optimal antiderivative.

$$
\int(d e x+c e)^{3}(a+b \operatorname{arcsinh}(d x+c))^{4} \mathrm{~d} x
$$

Optimal(type 3, 319 leaves, 16 steps):

$$
\begin{aligned}
& -\frac{45 b^{4} e^{3}(d x+c)^{2}}{128 d}+\frac{3 b^{4} e^{3}(d x+c)^{4}}{128 d}-\frac{45 b^{2} e^{3}(a+b \operatorname{arcsinh}(d x+c))^{2}}{128 d}-\frac{9 b^{2} e^{3}(d x+c)^{2}(a+b \operatorname{arcsinh}(d x+c))^{2}}{16 d} \\
& +\frac{3 b^{2} e^{3}(d x+c)^{4}(a+b \operatorname{arcsinh}(d x+c))^{2}}{16 d}-\frac{3 e^{3}(a+b \operatorname{arcsinh}(d x+c))^{4}}{32 d}+\frac{e^{3}(d x+c)^{4}(a+b \operatorname{arcsinh}(d x+c))^{4}}{4 d} \\
& +\frac{45 b^{3} e^{3}(d x+c)(a+b \operatorname{arcsinh}(d x+c)) \sqrt{1+(d x+c)^{2}}}{64 d}-\frac{3 b^{3} e^{3}(d x+c)^{3}(a+b \operatorname{arcsinh}(d x+c)) \sqrt{1+(d x+c)^{2}}}{32 d} \\
& +\frac{3 b e^{3}(d x+c)(a+b \operatorname{arcsinh}(d x+c))^{3} \sqrt{1+(d x+c)^{2}}}{8 d}-\frac{b e^{3}(d x+c)^{3}(a+b \operatorname{arcsinh}(d x+c))^{3} \sqrt{1+(d x+c)^{2}}}{4 d}
\end{aligned}
$$

Result(type 3, 682 leaves):
$\frac{1}{d}\left(\frac{(d x+c)^{4} e^{3} a^{4}}{4}+e^{3} b^{4}\left(\frac{(d x+c)^{2} \operatorname{arcsinh}(d x+c)^{4}\left(1+(d x+c)^{2}\right)}{4}-\frac{\operatorname{arcsinh}(d x+c)^{4}\left(1+(d x+c)^{2}\right)}{4}\right.\right.$
$-\frac{\operatorname{arcsinh}(d x+c)^{3}(d x+c)\left(1+(d x+c)^{2}\right)^{3 / 2}}{4}+\frac{5 \operatorname{arcsinh}(d x+c)^{3} \sqrt{1+(d x+c)^{2}}(d x+c)}{8}+\frac{5 \operatorname{arcsinh}(d x+c)^{4}}{32}$
$+\frac{3(d x+c)^{2}\left(1+(d x+c)^{2}\right) \operatorname{arcsinh}(d x+c)^{2}}{16}-\frac{3 \operatorname{arcsinh}(d x+c)(d x+c)\left(1+(d x+c)^{2}\right)^{3 / 2}}{32}$
$+\frac{51(d x+c) \operatorname{arcsinh}(d x+c) \sqrt{1+(d x+c)^{2}}}{64}+\frac{51 \operatorname{arcsinh}(d x+c)^{2}}{128}+\frac{3(d x+c)^{2}\left(1+(d x+c)^{2}\right)}{128}-\frac{3\left(1+(d x+c)^{2}\right) \operatorname{arcsinh}(d x+c)^{2}}{4}$
$\left.-\frac{3(d x+c)^{2}}{8}-\frac{3}{8}\right)+4 e^{3} a b^{3}\left(\frac{(d x+c)^{2} \operatorname{arcsinh}(d x+c)^{3}\left(1+(d x+c)^{2}\right)}{4}-\frac{\operatorname{arcsinh}(d x+c)^{3}\left(1+(d x+c)^{2}\right)}{4}\right.$
$-\frac{3 \operatorname{arcsinh}(d x+c)^{2}(d x+c)\left(1+(d x+c)^{2}\right)^{3 / 2}}{16}+\frac{15(d x+c) \operatorname{arcsinh}(d x+c)^{2} \sqrt{1+(d x+c)^{2}}}{32}+\frac{5 \operatorname{arcsinh}(d x+c)^{3}}{32}$
$+\frac{3 \operatorname{arcsinh}(d x+c)(d x+c)^{2}\left(1+(d x+c)^{2}\right)}{32}-\frac{3(d x+c)\left(1+(d x+c)^{2}\right)^{3 / 2}}{128}+\frac{51(d x+c) \sqrt{1+(d x+c)^{2}}}{256}+\frac{51 \operatorname{arcsinh}(d x+c)}{256}$

$$
\begin{aligned}
& \left.-\frac{3\left(1+(d x+c)^{2}\right) \operatorname{arcsinh}(d x+c)}{8}\right)+6 e^{3} a^{2} b^{2}\left(\frac{(d x+c)^{2}\left(1+(d x+c)^{2}\right) \operatorname{arcsinh}(d x+c)^{2}}{4}-\frac{\left(1+(d x+c)^{2}\right) \operatorname{arcsinh}(d x+c)^{2}}{4}\right. \\
& -\frac{\operatorname{arcsinh}(d x+c)(d x+c)\left(1+(d x+c)^{2}\right)^{3 / 2}}{8}+\frac{5(d x+c) \operatorname{arcsinh}(d x+c) \sqrt{1+(d x+c)^{2}}}{16}+\frac{5 \operatorname{arcsinh}(d x+c)^{2}}{32} \\
& \left.+\frac{(d x+c)^{2}\left(1+(d x+c)^{2}\right)}{32}-\frac{(d x+c)^{2}}{8}-\frac{1}{8}\right)+4 e^{3} a^{3} b\left(\frac{(d x+c)^{4} \operatorname{arcsinh}(d x+c)}{4}-\frac{(d x+c)^{3} \sqrt{1+(d x+c)^{2}}}{16}\right. \\
& \left.\left.+\frac{3(d x+c) \sqrt{1+(d x+c)^{2}}}{32}-\frac{3 \operatorname{arcsinh}(d x+c)}{32}\right)\right)
\end{aligned}
$$

Problem 43: Result more than twice size of optimal antiderivative.

$$
\int(d e x+c e)^{2}(a+b \operatorname{arcsinh}(d x+c))^{4} \mathrm{~d} x
$$

Optimal(type 3, 255 leaves, 13 steps):

$$
\begin{aligned}
& -\frac{160 b^{4} e^{2} x}{27}+\frac{8 b^{4} e^{2}(d x+c)^{3}}{81 d}-\frac{8 b^{2} e^{2}(d x+c)(a+b \operatorname{arcsinh}(d x+c))^{2}}{3 d}+\frac{4 b^{2} e^{2}(d x+c)^{3}(a+b \operatorname{arcsinh}(d x+c))^{2}}{9 d} \\
& \quad+\frac{e^{2}(d x+c)^{3}(a+b \operatorname{arcsinh}(d x+c))^{4}}{3 d}+\frac{160 b^{3} e^{2}(a+b \operatorname{arcsinh}(d x+c)) \sqrt{1+(d x+c)^{2}}}{27 d} \\
& \quad-\frac{8 b^{3} e^{2}(d x+c)^{2}(a+b \operatorname{arcsinh}(d x+c)) \sqrt{1+(d x+c)^{2}}}{27 d}+\frac{8 b e^{2}(a+b \operatorname{arcsinh}(d x+c))^{3} \sqrt{1+(d x+c)^{2}}}{9 d} \\
& \quad-\frac{4 b e^{2}(d x+c)^{2}(a+b \operatorname{arcsinh}(d x+c))^{3} \sqrt{1+(d x+c)^{2}}}{9 d}
\end{aligned}
$$

Result(type 3, 566 leaves):

$$
\begin{aligned}
& \frac{1}{d}\left(\frac{(d x+c)^{3} e^{2} a^{4}}{3}+e^{2} b^{4}\left(\frac{(d x+c) \operatorname{arcsinh}(d x+c)^{4}\left(1+(d x+c)^{2}\right)}{3}-\frac{\operatorname{arcsinh}(d x+c)^{4}(d x+c)}{3}\right.\right. \\
&-\frac{4(d x+c)^{2} \operatorname{arcsinh}(d x+c)^{3} \sqrt{1+(d x+c)^{2}}}{9}+\frac{8 \operatorname{arcsinh}(d x+c)^{3} \sqrt{1+(d x+c)^{2}}}{9}+\frac{4 \operatorname{arcsinh}(d x+c)^{2}(d x+c)\left(1+(d x+c)^{2}\right)}{9} \\
&-\frac{28(d x+c) \operatorname{arcsinh}(d x+c)^{2}}{9}-\frac{8 \operatorname{arcsinh}(d x+c)(d x+c)^{2} \sqrt{1+(d x+c)^{2}}}{27}+\frac{160 \operatorname{arcsinh}(d x+c) \sqrt{1+(d x+c)^{2}}}{27} \\
&\left.\quad+\frac{8(d x+c)\left(1+(d x+c)^{2}\right)}{81}-\frac{488 d x}{81}-\frac{488 c}{81}\right)+4 e^{2} a b^{3}\left(\frac{\operatorname{arcsinh}(d x+c)^{3}(d x+c)\left(1+(d x+c)^{2}\right)}{3}-\frac{\operatorname{arcsinh}(d x+c)^{3}(d x+c)}{3}\right. \\
&-\frac{\operatorname{arcsinh}(d x+c)^{2}(d x+c)^{2} \sqrt{1+(d x+c)^{2}}}{3}+\frac{2 \operatorname{arcsinh}(d x+c)^{2} \sqrt{1+(d x+c)^{2}}}{3}+\frac{2 \operatorname{arcsinh}(d x+c)(d x+c)\left(1+(d x+c)^{2}\right)}{9} \\
&\left.-\frac{14 \operatorname{arcsinh}(d x+c)(d x+c)}{9}-\frac{2(d x+c)^{2} \sqrt{1+(d x+c)^{2}}}{27}+\frac{40 \sqrt{1+(d x+c)^{2}}}{27}\right)+6 e^{2} a^{2} b^{2}\left(\frac{\operatorname{arcsinh}(d x+c)^{2}(d x+c)\left(1+(d x+c)^{2}\right)}{3}\right.
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{(d x+c) \operatorname{arcsinh}(d x+c)^{2}}{3}-\frac{2 \operatorname{arcsinh}(d x+c)(d x+c)^{2} \sqrt{1+(d x+c)^{2}}}{9}+\frac{4 \operatorname{arcsinh}(d x+c) \sqrt{1+(d x+c)^{2}}}{9}+\frac{2(d x+c)\left(1+(d x+c)^{2}\right)}{27} \\
& \left.\left.-\frac{14 d x}{27}-\frac{14 c}{27}\right)+4 e^{2} a^{3} b\left(\frac{(d x+c)^{3} \operatorname{arcsinh}(d x+c)}{3}-\frac{(d x+c)^{2} \sqrt{1+(d x+c)^{2}}}{9}+\frac{2 \sqrt{1+(d x+c)^{2}}}{9}\right)\right)
\end{aligned}
$$

Problem 44: Result more than twice size of optimal antiderivative.

$$
\int \frac{(a+b \operatorname{arcsinh}(d x+c))^{4}}{(d e x+c e)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 299 leaves, 13 steps):

$$
\begin{aligned}
& -\frac{(a+b \operatorname{arcsinh}(d x+c))^{4}}{d e^{2}(d x+c)}-\frac{8 b(a+b \operatorname{arcsinh}(d x+c))^{3} \operatorname{arctanh}\left(d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& -\frac{12 b^{2}(a+b \operatorname{arcsinh}(d x+c))^{2} \operatorname{polylog}\left(2,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}+\frac{12 b^{2}(a+b \operatorname{arcsinh}(d x+c))^{2} \operatorname{polylog}\left(2, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& \quad+\frac{24 b^{3}(a+b \operatorname{arcsinh}(d x+c)) \operatorname{polylog}\left(3,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}-\frac{24 b^{3}(a+b \operatorname{arcsinh}(d x+c)) \operatorname{polylog}\left(3, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& \quad-\frac{24 b^{4} \operatorname{polylog}\left(4,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}+\frac{24 b^{4} \operatorname{polylog}\left(4, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}
\end{aligned}
$$

Result(type 4, 819 leaves):

$$
\begin{aligned}
& -\frac{a^{4}}{d e^{2}(d x+c)}-\frac{b^{4} \operatorname{arcsinh}(d x+c)^{4}}{d e^{2}(d x+c)}-\frac{4 b^{4} \operatorname{arcsinh}(d x+c)^{3} \ln \left(1+d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& -\frac{12 b^{4} \operatorname{arcsinh}(d x+c)^{2} \operatorname{polylog}\left(2,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}+\frac{24 b^{4} \operatorname{arcsinh}(d x+c) \operatorname{polylog}\left(3,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& -\frac{24 b^{4} \operatorname{polylog}\left(4,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}+\frac{4 b^{4} \operatorname{arcsinh}(d x+c)^{3} \ln \left(1-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& \quad+\frac{12 b^{4} \operatorname{arcsinh}(d x+c)^{2} \operatorname{polylog}\left(2, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}-\frac{24 b^{4} \operatorname{arcsinh}(d x+c) \operatorname{polylog}\left(3, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& \left.\quad+\frac{24 b^{4} \operatorname{polylog}\left(4, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}-\frac{4 a b^{3} \operatorname{arcsinh}(d x+c)^{3}}{d e^{2}(d x}+c\right) \\
& \quad+\frac{24 a b^{3} \operatorname{arcsinh}(d x+c) \operatorname{polylog}\left(2, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}-\frac{24 a b^{3} \operatorname{polylog}\left(3, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& \quad-\frac{12 a b^{3} \operatorname{arcsinh}(d x+c)^{2} \ln \left(1+d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}-\frac{24 a b^{3} \operatorname{arcsinh}(d x+c) \operatorname{polylog}\left(2,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{24 a b^{3} \operatorname{polylog}\left(3,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}-\frac{6 a^{2} b^{2} \operatorname{arcsinh}(d x+c)^{2}}{d e^{2}(d x+c)}-\frac{12 a^{2} b^{2} \operatorname{arcsinh}(d x+c) \ln \left(1+d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& -\frac{12 a^{2} b^{2} \operatorname{polylog}\left(2,-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}+\frac{12 a^{2} b^{2} \operatorname{arcsinh}(d x+c) \ln \left(1-d x-c-\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}} \\
& +\frac{12 a^{2} b^{2} \operatorname{polylog}\left(2, d x+c+\sqrt{1+(d x+c)^{2}}\right)}{d e^{2}}-\frac{4 a^{3} b \operatorname{arcsinh}(d x+c)}{d e^{2}(d x+c)}-\frac{4 a^{3} b \operatorname{arctanh}\left(\frac{1}{\sqrt{1+(d x+c)^{2}}}\right)}{d e^{2}}
\end{aligned}
$$

Problem 49: Unable to integrate problem.

$$
\int(d e x+c e)^{2}(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \mathrm{~d} x
$$

Optimal(type 4, 265 leaves, 24 steps):
$\frac{e^{2}(d x+c)^{3}(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}{3 d}+\frac{b^{3 / 2} e^{2} \mathrm{e}^{\frac{3 a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{288 d}$

$-\frac{3 b^{3 / 2} e^{2} \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{32 d \mathrm{e}^{\frac{a}{b}}}+\frac{b e^{2} \sqrt{1+(d x+c)^{2}} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{3 d}$
$-\frac{b e^{2}(d x+c)^{2} \sqrt{1+(d x+c)^{2}} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{6 d}$
Result(type 8, 25 leaves):

$$
\int(d e x+c e)^{2}(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \mathrm{~d} x
$$

Problem 50: Unable to integrate problem.

$$
\int(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \mathrm{~d} x
$$

Optimal(type 4, 121 leaves, 9 steps):
$\frac{(d x+c)(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}{d}+\frac{3 b^{3 / 2} \mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{8 d}+\frac{3 b^{3 / 2} \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{8 d \mathrm{e}^{\frac{a}{b}}}$

$$
-\frac{3 b \sqrt{1+(d x+c)^{2}} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{2 d}
$$

Result(type 8, 14 leaves):

$$
\int(a+b \operatorname{arcsinh}(d x+c))^{3 / 2} \mathrm{~d} x
$$

Problem 52: Unable to integrate problem.

$$
\int(d e x+c e)^{2}(a+b \operatorname{arcsinh}(d x+c))^{7 / 2} \mathrm{~d} x
$$

Optimal(type 4, 398 leaves, 35 steps):
$-\frac{35 b^{2} e^{2}(d x+c)(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}{18 d}+\frac{35 b^{2} e^{2}(d x+c)^{3}(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}{108 d}+\frac{e^{2}(d x+c)^{3}(a+b \operatorname{arcsinh}(d x+c))^{7 / 2}}{3 d}$



$$
\begin{aligned}
& +\frac{7 b e^{2}(a+b \operatorname{arcsinh}(d x+c))^{5 / 2} \sqrt{1+(d x+c)^{2}}}{9 d}-\frac{7 b e^{2}(d x+c)^{2}(a+b \operatorname{arcsinh}(d x+c))^{5 / 2} \sqrt{1+(d x+c)^{2}}}{18 d} \\
& +\frac{175 b^{3} e^{2} \sqrt{1+(d x+c)^{2}} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{54 d}-\frac{35 b^{3} e^{2}(d x+c)^{2} \sqrt{1+(d x+c)^{2}} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{216 d}
\end{aligned}
$$

Result(type 8, 25 leaves):

$$
\int(d e x+c e)^{2}(a+b \operatorname{arcsinh}(d x+c))^{7 / 2} \mathrm{~d} x
$$

Problem 53: Unable to integrate problem.

$$
\int \frac{(d e x+c e)^{3}}{\sqrt{a+b \operatorname{arcsinh}(d x+c)}} \mathrm{d} x
$$

Optimal(type 4, 171 leaves, 15 steps):
$\frac{e^{3} \mathrm{e}^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{16 d \sqrt{b}}-\frac{e^{3} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{16 d \mathrm{e}^{\frac{2 a}{b}} \sqrt{b}}-\frac{e^{3} \mathrm{e}^{\frac{4 a}{b}} \operatorname{erf}\left(\frac{2 \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{32 d \sqrt{b}}$ $+\frac{e^{3} \operatorname{erfi}\left(\frac{2 \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{32 d \mathrm{e}^{\frac{4 a}{b}} \sqrt{b}}$
Result(type 8, 25 leaves):

$$
\int \frac{(d e x+c e)^{3}}{\sqrt{a+b \operatorname{arcsinh}(d x+c)}} \mathrm{d} x
$$

Problem 54: Unable to integrate problem.

$$
\int \frac{d e x+c e}{\sqrt{a+b \operatorname{arcsinh}(d x+c)}} \mathrm{d} x
$$

Optimal(type 4, 87 leaves, 10 steps):

$$
-\frac{e \mathrm{e}^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{8 d \sqrt{b}}+\frac{e \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{8 d \mathrm{e}^{\frac{2 a}{b}} \sqrt{b}}
$$

Result(type 8, 23 leaves):

$$
\int \frac{d e x+c e}{\sqrt{a+b \operatorname{arcsinh}(d x+c)}} \mathrm{d} x
$$

Problem 55: Unable to integrate problem.


Optimal(type 4, 71 leaves, 7 steps):


Result(type 8, 14 leaves):

$$
\int \frac{1}{\sqrt{a+b \operatorname{arcsinh}(d x+c)}} \mathrm{d} x
$$

Problem 56: Unable to integrate problem.

$$
\int \frac{(d e x+c e)^{3}}{(a+b \operatorname{arcsinh}(d x+c))^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 272 leaves, 26 steps):


$$
-\frac{e^{3} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{3 b^{5 / 2} d \mathrm{e}^{\frac{2 a}{b}}}-\frac{2 e^{3}(d x+c)^{3} \sqrt{1+(d x+c)^{2}}}{3 b d(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}-\frac{4 e^{3}(d x+c)^{2}}{b^{2} d \sqrt{a+b \operatorname{arcsinh}(d x+c)}}
$$

$$
-\frac{16 e^{3}(d x+c)^{4}}{3 b^{2} d \sqrt{a+b \operatorname{arcsinh}(d x+c)}}
$$

Result(type 8, 25 leaves):

$$
\int \frac{(d e x+c e)^{3}}{(a+b \operatorname{arcsinh}(d x+c))^{5 / 2}} \mathrm{~d} x
$$

Problem 57: Unable to integrate problem.

$$
\int \frac{1}{(a+b \operatorname{arcsinh}(d x+c))^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 127 leaves, 9 steps):
$\frac{2 \mathrm{e}^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{3 b^{5 / 2} d}+\frac{2 \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{3 b^{5 / 2} d \mathrm{e}^{\frac{a}{b}}}-\frac{2 \sqrt{1+(d x+c)^{2}}}{3 b d(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}-\frac{4(d x+c)}{3 b^{2} d \sqrt{a+b \operatorname{arcsinh}(d x+c)}}$
Result(type 8, 14 leaves):

$$
\int \frac{1}{(a+b \operatorname{arcsinh}(d x+c))^{5 / 2}} \mathrm{~d} x
$$

Problem 59: Unable to integrate problem.

$$
\int \frac{d e x+c e}{(a+b \operatorname{arcsinh}(d x+c))^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 210 leaves, 11 steps):
$-\frac{4 e}{15 b^{2} d(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}-\frac{8 e(d x+c)^{2}}{15 b^{2} d(a+b \operatorname{arcsinh}(d x+c))^{3 / 2}}+\frac{8 e \mathrm{e}^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{15 b^{7 / 2} d}$

$$
+\frac{8 e \mathrm{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arcsinh}(d x+c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{15 b^{7 / 2} d \mathrm{e}^{\frac{2 a}{b}}}-\frac{2 e(d x+c) \sqrt{1+(d x+c)^{2}}}{5 b d(a+b \operatorname{arcsinh}(d x+c))^{5 / 2}}-\frac{32 e(d x+c) \sqrt{1+(d x+c)^{2}}}{15 b^{3} d \sqrt{a+b \operatorname{arcsinh}(d x+c)}}
$$

Result(type 8, 23 leaves):

$$
\int \frac{d e x+c e}{(a+b \operatorname{arcsinh}(d x+c))^{7 / 2}} \mathrm{~d} x
$$

Problem 64: Unable to integrate problem.

$$
\int(d e x+c e)^{3 / 2}(a+b \operatorname{arcsinh}(d x+c))^{2} \mathrm{~d} x
$$

Optimal(type 5, 110 leaves, 3 steps):
$\frac{2(e(d x+c))^{5 / 2}(a+b \operatorname{arcsinh}(d x+c))^{2}}{5 d e}-\frac{8 b(e(d x+c))^{7 / 2}(a+b \operatorname{arcsinh}(d x+c)) \text { hypergeom }\left(\left[\frac{1}{2}, \frac{7}{4}\right],\left[\frac{11}{4}\right],-(d x+c)^{2}\right)}{35 d e^{2}}$

$$
+\frac{16 b^{2}(e(d x+c))^{9 / 2} \text { HypergeometricPFQ }\left(\left[1, \frac{9}{4}, \frac{9}{4}\right],\left[\frac{11}{4}, \frac{13}{4}\right],-(d x+c)^{2}\right)}{315 d e^{3}}
$$

Result(type 8, 25 leaves):

$$
\int(d e x+c e)^{3 / 2}(a+b \operatorname{arcsinh}(d x+c))^{2} \mathrm{~d} x
$$

Problem 65: Unable to integrate problem.

$$
\int(a+b \operatorname{arcsinh}(d x+c))^{2} \sqrt{d e x+c e} \mathrm{~d} x
$$

Optimal(type 5, 110 leaves, 3 steps):
$\frac{2(e(d x+c))^{3 / 2}(a+b \operatorname{arcsinh}(d x+c))^{2}}{3 d e}-\frac{8 b(e(d x+c))^{5 / 2}(a+b \operatorname{arcsinh}(d x+c)) \text { hypergeom }\left(\left[\frac{1}{2}, \frac{5}{4}\right],\left[\frac{9}{4}\right],-(d x+c)^{2}\right)}{15 d e^{2}}$

$$
+\frac{16 b^{2}(e(d x+c))^{7 / 2} \text { HypergeometricPFQ }\left(\left[1, \frac{7}{4}, \frac{7}{4}\right],\left[\frac{9}{4}, \frac{11}{4}\right],-(d x+c)^{2}\right)}{105 d e^{3}}
$$

Result(type 8, 25 leaves):

$$
\int(a+b \operatorname{arcsinh}(d x+c))^{2} \sqrt{d e x+c e} \mathrm{~d} x
$$

Problem 66: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(d x+c))^{2}}{(d e x+c e)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 110 leaves, 3 steps):
$-\frac{2(a+b \operatorname{arcsinh}(d x+c))^{2}}{3 d e(e(d x+c))^{3 / 2}}-\frac{8 b(a+b \operatorname{arcsinh}(d x+c)) \text { hypergeom }\left(\left[-\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{4}\right],-(d x+c)^{2}\right)}{3 d e^{2} \sqrt{e(d x+c)}}$
$+\frac{16 b^{2} \text { HypergeometricPFQ }\left(\left[\frac{1}{4}, \frac{1}{4}, 1\right],\left[\frac{3}{4}, \frac{5}{4}\right],-(d x+c)^{2}\right) \sqrt{e(d x+c)}}{3 d e^{3}}$
Result(type 8, 25 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(d x+c))^{2}}{(d e x+c e)^{5 / 2}} \mathrm{~d} x
$$

Problem 67: Unable to integrate problem.

$$
\int \frac{(a+b \operatorname{arcsinh}(d x+c))^{2}}{(d e x+c e)^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 110 leaves, 3 steps):
$-\frac{2(a+b \operatorname{arcsinh}(d x+c))^{2}}{5 d e(e(d x+c))^{5 / 2}}-\frac{8 b(a+b \operatorname{arcsinh}(d x+c)) \text { hypergeom }\left(\left[-\frac{3}{4}, \frac{1}{2}\right],\left[\frac{1}{4}\right],-(d x+c)^{2}\right)}{15 d e^{2}(e(d x+c))^{3 / 2}}$
$-16 b^{2}$ HypergeometricPFQ $\left(\left[-\frac{1}{4},-\frac{1}{4}, 1\right],\left[\frac{1}{4}, \frac{3}{4}\right],-(d x+c)^{2}\right)$

$$
15 d e^{3} \sqrt{e(d x+c)}
$$

Result(type 8, 25 leaves):

$$
\int \frac{(a+b \operatorname{arcsinh}(d x+c))^{2}}{(d e x+c e)^{7 / 2}} \mathrm{~d} x
$$

[^2]$$
\int\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)^{3 / 2} \operatorname{arcsinh}(b x+a)^{2} \mathrm{~d} x
$$

Optimal(type 3, 165 leaves, 11 steps):
$\frac{(b x+a)\left(1+(b x+a)^{2}\right)^{3 / 2}}{32 b}-\frac{9 \operatorname{arcsinh}(b x+a)}{64 b}-\frac{3(b x+a)^{2} \operatorname{arcsinh}(b x+a)}{8 b}-\frac{\left(1+(b x+a)^{2}\right)^{2} \operatorname{arcsinh}(b x+a)}{8 b}$
$+\frac{(b x+a)\left(1+(b x+a)^{2}\right)^{3 / 2} \operatorname{arcsinh}(b x+a)^{2}}{4 b}+\frac{\operatorname{arcsinh}(b x+a)^{3}}{8 b}+\frac{15(b x+a) \sqrt{1+(b x+a)^{2}}}{64 b}$

$$
+\frac{3(b x+a) \operatorname{arcsinh}(b x+a)^{2} \sqrt{1+(b x+a)^{2}}}{8 b}
$$

Result(type 3, 478 leaves):
$\frac{1}{64 b}\left(16 \operatorname{arcsinh}(b x+a)^{2} \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} x^{3} b^{3}-8 \operatorname{arcsinh}(b x+a) x^{4} b^{4}+48 \operatorname{arcsinh}(b x+a)^{2} \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} x^{2} a b^{2}-32 \operatorname{arcsinh}(b x\right.$
+a) $x^{3} a b^{3}+48 \operatorname{arcsinh}(b x+a)^{2} \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} x a^{2} b+2 \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} x^{3} b^{3}-48 \operatorname{arcsinh}(b x+a) x^{2} a^{2} b^{2}+16 \operatorname{arcsinh}(b x$
$+a)^{2} \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} a^{3}+6 \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} x^{2} a b^{2}-32 \operatorname{arcsinh}(b x+a) x a^{3} b+40 \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} \operatorname{arcsinh}(b x+a)^{2} x b$
$+6 \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} x a^{2} b-40 \operatorname{arcsinh}(b x+a) x^{2} b^{2}-8 \operatorname{arcsinh}(b x+a) a^{4}+40 \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} \operatorname{arcsinh}(b x+a)^{2} a$
$+2 \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} a^{3}-80 \operatorname{arcsinh}(b x+a) x a b+17 \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} x b+8 \operatorname{arcsinh}(b x+a)^{3}-40 \operatorname{arcsinh}(b x+a) a^{2}$
$\left.+17 \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1} a-17 \operatorname{arcsinh}(b x+a)\right)$

Problem 82: Unable to integrate problem.

$$
\int x \operatorname{arcsinh}\left(a x^{n}\right) \mathrm{d} x
$$

Optimal(type 5, 53 leaves, 3 steps):

$$
\frac{x^{2} \operatorname{arcsinh}\left(a x^{n}\right)}{2}-\frac{a n x^{2+n} \text { hypergeom }\left(\left[\frac{1}{2}, \frac{2+n}{2 n}\right],\left[\frac{3}{2}+\frac{1}{n}\right],-a^{2} x^{2 n}\right)}{2(2+n)}
$$

Result(type 8, 10 leaves):

$$
\int x \operatorname{arcsinh}\left(a x^{n}\right) \mathrm{d} x
$$

Problem 83: Unable to integrate problem.

$$
\int \frac{\operatorname{arcsinh}\left(a x^{n}\right)}{x^{2}} \mathrm{~d} x
$$

Optimal(type 5, 57 leaves, 3 steps):

$$
-\frac{\operatorname{arcsinh}\left(a x^{n}\right)}{x}-\frac{a n x^{-1+n} \text { hypergeom }\left(\left[\frac{1}{2}, \frac{-1+n}{2 n}\right],\left[\frac{3}{2}-\frac{1}{2 n}\right],-a^{2} x^{2 n}\right)}{1-n}
$$

Result(type 8, 12 leaves):

$$
\int \frac{\operatorname{arcsinh}\left(a x^{n}\right)}{x^{2}} \mathrm{~d} x
$$

Problem 84: Unable to integrate problem.

$$
\int\left(a+b \operatorname{arcsinh}\left(\mathrm{I}+d x^{2}\right)\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 69 leaves, 2 steps):

$$
8 b^{2} x+x\left(a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)\right)^{2}-\frac{4 b\left(a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)\right) \sqrt{2 \mathrm{I} d x^{2}+d^{2} x^{4}}}{d x}
$$

Result(type 8, 17 leaves):

$$
\int\left(a+b \operatorname{arcsinh}\left(\mathrm{I}+d x^{2}\right)\right)^{2} \mathrm{~d} x
$$

Problem 85: Unable to integrate problem.

$$
\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(-\mathrm{I}+d x^{2}\right)\right)^{3}} \mathrm{~d} x
$$

Optimal(type 4, 226 leaves, 2 steps):

$$
\begin{aligned}
& -\frac{x}{8 b^{2}\left(a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)\right)}+\frac{x \operatorname{Shi}\left(\frac{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}{2 b}\right)\left(\cosh \left(\frac{a}{2 b}\right)+\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right)}{16 b^{3}\left(\cos \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)-\sin \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)\right)} \\
& -\frac{x \operatorname{Ci}\left(\frac{\frac{\mathrm{I}}{2}\left(a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)\right)}{b}\right)\left(\mathrm{I} \cosh \left(\frac{a}{2 b}\right)+\sinh \left(\frac{a}{2 b}\right)\right)}{16 b^{3}\left(\cos \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)-\sin \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)\right)}-\frac{\sqrt{-2 \mathrm{I} d x^{2}+d^{2} x^{4}}}{4 b d x\left(a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)\right)^{2}}
\end{aligned}
$$

Result(type 8, 17 leaves):

$$
\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(-\mathrm{I}+d x^{2}\right)\right)^{3}} \mathrm{~d} x
$$

Problem 86: Unable to integrate problem.

$$
\int \sqrt{a+b \operatorname{arcsinh}\left(\mathrm{I}+d x^{2}\right)} \mathrm{d} x
$$

Optimal(type 4, 205 leaves, 1 step):
$x$ FresnelS $\left(\frac{\sqrt{\frac{-\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}}{\sqrt{\pi}}\right)\left(\cosh \left(\frac{a}{2 b}\right)+\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}$

$$
\left(\cos \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)+\sin \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)\right) \sqrt{\frac{-\mathrm{I}}{b}}
$$

$$
-\frac{b x \text { FresnelC }\left(\frac{\sqrt{\frac{-\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}}{\sqrt{\pi}}\right)\left(\mathrm{I} \cosh \left(\frac{a}{2 b}\right)+\sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\frac{-\mathrm{I}}{b}} \sqrt{\pi}}{\cos \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)+\sin \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)}+x \sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}
$$

Result(type 8, 17 leaves):

$$
\int \sqrt{a+b \operatorname{arcsinh}\left(\mathrm{I}+d x^{2}\right)} \mathrm{d} x
$$

Problem 87: Unable to integrate problem.

$$
\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(\mathrm{I}+d x^{2}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 230 leaves, 1 step):

$$
\begin{aligned}
& -\frac{\left(\frac{-\mathrm{I}}{b}\right)^{3 / 2} x \text { FresnelC }\left(\frac{\sqrt{\frac{-\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}}{\sqrt{\pi}}\right)\left(\cosh \left(\frac{a}{2 b}\right)-\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}}{\cos \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)+\sin \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)} \\
& +\frac{\left(\frac{-\mathrm{I}}{b}\right)^{3 / 2} x \operatorname{FresnelS}\left(\frac{\sqrt{\frac{-\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}}{\cos \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)+\sin \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)}\right.}{}+\frac{\sqrt{\pi}}{2}-\frac{a}{b d x \sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}}
\end{aligned}
$$

Result(type 8, 17 leaves):

$$
\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(\mathrm{I}+d x^{2}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 88: Unable to integrate problem.

$$
\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(\mathrm{I}+d x^{2}\right)\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 253 leaves, 2 steps):
$-\frac{x \text { FresnelS }\left(\frac{\sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}}{\sqrt{\mathrm{I} b} \sqrt{\pi}}\right)\left(\cosh \left(\frac{a}{2 b}\right)-\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}}{3 b^{2}\left(\cos \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)+\sin \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)\right) \sqrt{\mathrm{I} b}}$
$-\frac{x \text { FresnelC }\left(\frac{\sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}}{\sqrt{\mathrm{I} b} \sqrt{\pi}}\right)\left(\cosh \left(\frac{a}{2 b}\right)+\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}}{3 b^{2}\left(\cos \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)+\sin \left(\frac{\arcsin \left(-1+\mathrm{I} d x^{2}\right)}{2}\right)\right) \sqrt{\mathrm{I} b}}-\frac{\sqrt{2 \mathrm{I} d x^{2}+d^{2} x^{4}}}{3 b d x\left(a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)\right)^{3 / 2}}$
$-\frac{x}{3 b^{2} \sqrt{a-\mathrm{I} b \arcsin \left(-1+\mathrm{I} d x^{2}\right)}}$
Result(type 8, 17 leaves):


Problem 89: Unable to integrate problem.

$$
\int\left(a+b \operatorname{arcsinh}\left(-\mathrm{I}+d x^{2}\right)\right)^{5 / 2} \mathrm{~d} x
$$

Optimal(type 4, 283 leaves, 2 steps):
$x\left(a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)\right)^{5 / 2}+\frac{15 b^{2} x \text { FresnelS }\left(\frac{\sqrt{\frac{\mathrm{I}}{b} \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}}}{\sqrt{\pi}}\right)\left(\cosh \left(\frac{a}{2 b}\right)-\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}}{\left(\cos \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)-\sin \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)\right) \sqrt{\frac{\mathrm{I}}{b}}}$
$-\frac{15 b^{2} x \text { FresnelC }\left(\frac{\sqrt{\frac{\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}}{\sqrt{\pi}}\right)\left(\cosh \left(\frac{a}{2 b}\right)+\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}}{\left(\cos \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)-\sin \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)\right) \sqrt{\frac{\mathrm{I}}{b}}}-\frac{5 b\left(a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)\right)^{3 / 2} \sqrt{-2 \mathrm{I} d x^{2}+d^{2} x^{4}}}{d x}$

$$
+15 b^{2} x \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}
$$

Result(type 8, 17 leaves):

$$
\int\left(a+b \operatorname{arcsinh}\left(-\mathrm{I}+d x^{2}\right)\right)^{5 / 2} \mathrm{~d} x
$$

Problem 90: Unable to integrate problem.

$$
\int \sqrt{a+b \operatorname{arcsinh}\left(-\mathrm{I}+d x^{2}\right)} \mathrm{d} x
$$

Optimal (type 4, 208 leaves, 1 step):
$\frac{x \text { FresnelS }\left(\frac{\sqrt{\frac{\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}}{\sqrt{\pi}}\right)\left(\cosh \left(\frac{a}{2 b}\right)-\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}}{\left(\cos \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)-\sin \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)\right) \sqrt{\frac{\mathrm{I}}{b}}}$
$-\frac{x \text { FresnelC }\left(\frac{\sqrt{\frac{\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}}{\left(\cos \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)-\sin \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)\right) \sqrt{\frac{\mathrm{I}}{b}}}+x \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}\right.}{\left(\frac{\left.\cosh \left(\frac{a}{2 b}\right)+\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}}{2}\right)+x(1)}$
Result (type 8, 17 leaves):

$$
\int \sqrt{a+b \operatorname{arcsinh}\left(-\mathrm{I}+d x^{2}\right)} \mathrm{d} x
$$

Problem 91: Unable to integrate problem.

$$
\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(-\mathrm{I}+d x^{2}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 234 leaves, 1 step):

$$
\begin{aligned}
& \frac{\left(\frac{\mathrm{I}}{b}\right)^{3 / 2} x \text { FresnelS }\left(\frac{\sqrt{\frac{\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}}{\sqrt{\pi}}\right)\left(\cosh \left(\frac{a}{2 b}\right)-\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi}}{\cos \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)-\sin \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)} \\
& -\frac{\left(\frac{\mathrm{I}}{b}\right)^{3 / 2} x \text { FresnelC }\left(\frac{\sqrt{\frac{\mathrm{I}}{b}} \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}}{\cos \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)-\sin \left(\frac{\arcsin \left(1+\mathrm{I} d x^{2}\right)}{2}\right)}\right.}{} \quad \begin{array}{l}
\left.\cosh \left(\frac{a}{2 b}\right)+\mathrm{I} \sinh \left(\frac{a}{2 b}\right)\right) \sqrt{\pi} \\
-\frac{\sqrt{-2 \mathrm{I} d x^{2}+d^{2} x^{4}}}{b d x \sqrt{a-\mathrm{I} b \arcsin \left(1+\mathrm{I} d x^{2}\right)}}
\end{array}
\end{aligned}
$$

Result (type 8, 17 leaves):

$$
\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(-\mathrm{I}+d x^{2}\right)\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 92: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(a+b \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)\right)^{3}}{-c^{2} x^{2}+1} d x
$$

Optimal(type 4, 285 leaves, 8 steps):

$$
-\frac{\left(a+b \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)\right)^{4}}{4 b c}-\frac{\left(a+b \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)\right)^{3} \ln \left(1-\frac{1}{\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+\sqrt{1+\frac{-c x+1}{c x+1}}\right)^{2}}\right)}{c}
$$

$$
+\frac{3 b\left(a+b \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)\right)^{2} \operatorname{polylog}\left(2, \frac{1}{\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+\sqrt{1+\frac{-c x+1}{c x+1}}\right)^{2}}\right)}{2 c}
$$

$$
+\frac{3 b^{2}\left(a+b \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)\right) \operatorname{polylog}\left(3, \frac{1}{\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+\sqrt{\left.1+\frac{-c x+1}{c x+1}\right)^{2}}\right)}\right.}{2 c}+\frac{3 b^{3} \operatorname{polylog}\left(4, \frac{1}{\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+\sqrt{1+\frac{-c x+1}{c x+1}}\right)^{2}}\right)}{4 c}
$$

Result(type 4, 1174 leaves):

$-\frac{3 b^{3} \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^{2} \text { polylog }\left(2,-\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-\sqrt{1+\frac{-c x+1}{c x+1}}\right)}{c}$
$+\frac{6 b^{3} \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right) \operatorname{polylog}\left(3,-\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-\sqrt{1+\frac{-c x+1}{c x+1}}\right)}{c}-\frac{6 b^{3} \operatorname{polylog}\left(4,-\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-\sqrt{1+\frac{-c x+1}{c x+1}}\right)}{c}$

$$
-\frac{b^{3} \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^{3} \ln \left(1-\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}-\sqrt{1+\frac{-c x+1}{c x+1}}\right)}{-}-3 b^{3} \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^{2} \operatorname{polylog}\left(2, \frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+\sqrt{1+\frac{-c x+1}{c x+1}}\right)
$$

$$
+\frac{6 b^{3} \operatorname{arcsinh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right) \text { polylog }\left(3, \frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+\sqrt{1+\frac{-c x+1}{c x+1}}\right)}{\left.\left(-\frac{6 b^{3} \operatorname{polylog}\left(4, \frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+\sqrt{1+\frac{-c x+1}{c x+1}}\right)}{c}\right) . \sqrt{c}\right)}
$$



Problem 97: Result more than twice size of optimal antiderivative.

$$
\int \frac{b x+a+\sqrt{1+(b x+a)^{2}}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 3, 91 leaves, 9 steps):

$$
-\frac{a}{x}+b \operatorname{arcsinh}(b x+a)+b \ln (x)-\frac{a b \operatorname{arctanh}\left(\frac{a b x+a^{2}+1}{\sqrt{a^{2}+1} \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}}\right)}{\sqrt{a^{2}+1}}-\frac{\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}}{x}
$$

Result(type 3, 266 leaves):

$$
-\frac{\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)^{3 / 2}}{\left(a^{2}+1\right) x}+\frac{2 a b \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}}{a^{2}+1}+\frac{a^{2} b^{2} \ln \left(\frac{b^{2} x+a b}{\sqrt{b^{2}}}+\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}\right)}{\left(a^{2}+1\right) \sqrt{b^{2}}}
$$



Problem 100: Unable to integrate problem.

$$
\int \frac{x}{\operatorname{arcsinh}(\sinh (x))} \mathrm{d} x
$$

Optimal(type 3, 25 leaves, ? steps):

$$
\operatorname{arcsinh}(\sinh (x))+\ln (\operatorname{arcsinh}(\sinh (x)))\left(-\operatorname{arcsinh}(\sinh (x))+x \operatorname{sech}(x) \sqrt{\cosh (x)^{2}}\right)
$$

Result(type 8, 9 leaves):

$$
\int \frac{x}{\operatorname{arcsinh}(\sinh (x))} \mathrm{d} x
$$

Summary of Integration Test Results
324 integration problems


A - 182 optimal antiderivatives
B - 49 more than twice size of optimal antiderivatives
C - 1 unnecessarily complex antiderivatives
D - 92 unable to integrate problems
E - O integration timeouts


[^0]:    Problem 147: Unable to integrate problem.

[^1]:    Problem 173: Unable to integrate problem.

[^2]:    Problem 73: Result more than twice size of optimal antiderivative.

